

ISOSTATIC REBOUND IN THE LAKE AGASSIZ BASIN  
SINCE THE LATE WISCONSINAN

by

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This thesis, submitted by Eric C. Brevik in partial fulfillment of the requirements for the degree of Master of Arts from the University of North Dakota, has been read by the Faculty Advisory Committee under whom the work has been done and is hereby approved.

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## ABSTRACT

This study addressed three main questions: 1) how thick was the ice that covered the southern Lake Agassiz basin during the Wisconsinan and how much that ice depressed the crust, 2) how much rebound has occurred since deglaciation and whether or not rebound is complete, and 3) what were the effects of this rebound on the basin.

The most direct method of measuring rebound in the Lake Agassiz basin is from strandlines left by glacial Lake Agassiz. The oldest complete strandline, the Herman, presumably rebounded, with the northern end rebounding more because the ice was thicker there and had melted from that end later. The difference in elevation of this strandline represents absolute minimum rebound, 54.5 meters. Up to 73% of rebound was restrained; the initial depression may have been as much as 200 meters. However, restrained rebound may have been retarded as ice was replaced by Lake Agassiz water and sediments. The average depth of Lake Agassiz at Grand Forks, ND, was as much as 100 meters, and the average thickness of sediments as much as 46 meters. These masses would cause crustal depression of 38 meters and 40 meters, respectively. The sediments are still in place in the Lake Agassiz Basin, causing 40 meters of depression. When added to the 54.5 meters of minimum depression, a total of 94.5 meters of depression is indicated. Minimum ice thickness would have been approximately 280 meters. Using a slope profile method, former ice thickness in the

Grand Forks, ND area was about 390 meters, with approximately 424 meters at the international border. Basal shear stress methods indicate ice thicknesses between 313 and 986 m. Maximum ice thickness indicated by the strandlines is 1040 meters.

Ice thickness must have exceeded the minimum. Several beach and scarp remnants are as much as 30 meters above the Herman strandline. On the other hand, the water and sediments of Lake Agassiz slowed rebound. Ice thickness, therefore, most likely was between 435 and 986 meters, causing a depression of 140 to 330 m.

Results of the rebound include decreased river gradients, changing river courses, and more frequent and intense flooding in the Lake Agassiz basin. Rebound definitely continues north of Lake Winnipeg, and may still be occurring in the southern Lake Agassiz basin, although the strandlines indicate that rebound in the southern Lake Agassiz basin is complete. In either case, the potential for increased flooding exists.

## INTRODUCTION

### General

The Lake Agassiz basin (Red River Valley) formed about 11,600 years B.P., when this region was submerged beneath Glacial Lake Agassiz. Along the former shorelines of Lake Agassiz, a series of beach ridges and erosional strandlines formed. Today, these features are tilted, decreasing in elevation from north to south. Because such features form at the water line, the shoreline of a lake should be at a constant elevation. These now-tilted strandlines are the strongest direct evidence that the crust in this area has rebounded since the draining of Lake Agassiz. The purpose of this study is to determine how much rebound has occurred in the southern Lake Agassiz basin and how this rebound has effected the basin.

Questions posed in this study have significant value to the inhabitants of several cities along the Red River of the North, which is in the Lake Agassiz basin. The Red River flows north, and tilted Lake Agassiz strandlines indicate that more rebound has occurred in the northern end of the basin since drainage of the lake. In other words, the gradient of the northward flowing river has decreased over time. This decreased gradient has led to changes in the river and its tributary system, changes that have and will continue to affect the people living along the river. Part of understanding these changes, and making sound decisions concerning them, hinges on

understanding the mechanism driving them. That mechanism is isostatic rebound, crustal uplift that occurs in order to maintain equilibrium of the earth's crust.

### Purpose

The purpose of this study was to: 1) determine the maximum ice thickness in the Lake Agassiz Basin during the late Wisconsinan stage and calculate the amount of crustal depression that resulted from that ice; 2) calculate the amount of crustal rebound that has taken place since melting of the ice; 3) determine how much, if any, residual rebound remains; and 4) examine some of the changes that have occurred in the Red River Valley as a result of the rebound and determine what effects those changes might have on the current occupants of the basin.

### Location

This study concentrates on the evidence for post-glacial rebound in the Lake Agassiz Basin of North Dakota, with particular reference to the Grand Forks area (Figure 1).

### Previous Work

#### General

The surficial deposits in this area were first recognized as resulting from a glacial lake by W. H. Keating (1825). Many early researchers believed that the former lake existed because of a moraine dam, but, according to Elson (1983), in 1872

Figure 1 - The Lake Agassiz Basin of North Dakota, as marked by Lake Agassiz's highest strandline (Modified from Bluemle, 1991, p 77)

Winchell became the first to suggest that the lake had been dammed by the retreating ice mass. Then, in 1879, Warren Upham began to map the lake deposits and named the former lake "glacial Lake Agassiz". Upham (1896) later published a classic report on the evidence for and characteristics of glacial Lake Agassiz.

The hypothesis that glacial rebound caused the southward slope of the Lake Agassiz strandlines was first proposed by T. F. Jamieson in 1865, and a modified version of this theory was adopted by Upham (Elson, 1983). Johnston (1946) subsequently traced Lake Agassiz strandlines as far north as Saskatchewan's Pasquia Hills and published his interpretation of glacial rebound in the Lake Agassiz area, a paper that still stands as the basis for all subsequent studies on Lake Agassiz shoreline deformation.

Although numerous subsequent theses and papers (Biek, 1993; Bluemle, 1991b, p 80-82; Teller and Bluemle, 1983; Kupsch, 1967) allude to post-glacial rebound in this region of North Dakota and Minnesota, no additional research on the amount of post-glacial rebound has been published.

#### Geology of the Area

The Lake Agassiz Basin is underlain by a sequence of till and lacustrine sand, silt, and clay units that vary in thickness. These sediments are, in turn, underlain by three bedrock lithologies (Figures 2 & 3). The first is the Precambrian rocks of the Canadian Shield, which form the bedrock in the northern parts of Minnesota. These

Figure 2 - Cross-section at the international border between North Dakota and Manitoba, showing the geologic age of the bedrock of the Lake Agassiz Basin. The names of the rock units can be found on Figure 3 (Teller and Bluemle, 1983, p 10).

Figure 3 - Stratigraphic column showing the units in the Lake Agassiz Basin. Note that on Figure 2, the geologic ages of the units are shown. On this stratigraphic column, the names of the units are given as well as the ages (Teller and Bluemle, 1983, p 14).



crystalline rocks are typically granitic and highly metamorphosed (Bluemle, 1973; Teller and Bluemle, 1983).

Along the axis of the present-day Red River Valley, the bedrock is mainly Paleozoic carbonates (Teller and Bluemle, 1983). In the Grand Forks area bedrock is dominated by Ordovician dolostones, namely the Red River and Stony Mountain Formations (Hansen and Kume, 1970, p 10-13).

Finally, the western edge of the Lake Agassiz Basin is underlain by Mesozoic shales (Teller and Bluemle, 1983), primarily the Pierre Shale (Cretaceous) at the Pembina escarpment (Figure 2).

### Glacial History

The Lake Agassiz Basin in North Dakota experienced numerous advances and subsequent retreats by various lobes of the Laurentide Ice Sheet. The basin probably contained lakes during each advance and retreat, but evidence of these early lakes has not been found (Fenton et al., 1983).

Approximately 20,000 C<sup>14</sup>\* years B.P., ice from the Keewatin center advanced as far south as central Iowa. It then began to retreat (Figure 4a), presumably as far as the Lake Agassiz basin. This was followed by a readvance at about 17,000 C<sup>14</sup> years B.P., and another advance about 14,000 C<sup>14</sup> years B.P. A fourth readvance occurred at about 12,300 C<sup>14</sup> years B.P. (Clayton and Moran, 1982). As this lobe retreated, the Dunvilla Formation was deposited in the Minnesota portion of the Lake Agassiz Basin, representing the first evidence of a proglacial lake in the basin (Fenton et al., 1983).

Figure 4:

A - Glacier extent in the Lake Agassiz Basin approximately 20,000 years B.P.  
(Fenton et al<sub>M</sub> 1983, p 60)

B - Glacier extent in the Lake Agassiz Basin approximately 11,700 years B.P.,  
showing earliest Lake Agassiz (Fenton et al., 1983, p 62).

C - Lake Agassiz approximately 11,200 years B.P., showing the final time that  
ice advanced into present day North Dakota (Fenton et al., 1983, p 65).

Three additional advances occurred before the earliest recognized phase of Lake Agassiz at approximately 11,700 C<sup>14</sup> years B.P. (Figure 4b) (Fenton et al., 1983, p. 61).

After Lake Agassiz had formed, the ice advanced again, about 11,200 C<sup>14</sup> years B.P., but remained within the Lake Agassiz basin. This advance deposited the Marchand Formation in northeastern North Dakota and northwestern Minnesota, and signaled the final time ice existed south of the present international border in the Lake Agassiz basin (Figure 4c) (Fenton et al., 1983).

#### Lake Agassiz Phases

The earliest phase of Lake Agassiz was the Cass Phase, which lasted from approximately 11,700 to 11,600 C<sup>14</sup> years B.P. Lake Agassiz was just beginning to form as the ice to the north blocked drainage in the basin. The Herman strandline was formed at this time; the lake drained primarily through the Minnesota River Valley (Fenton et al., 1983).

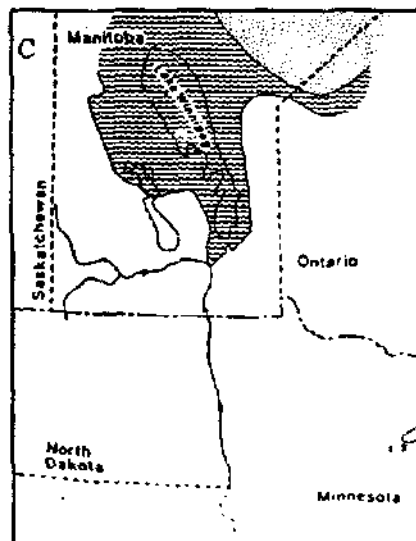
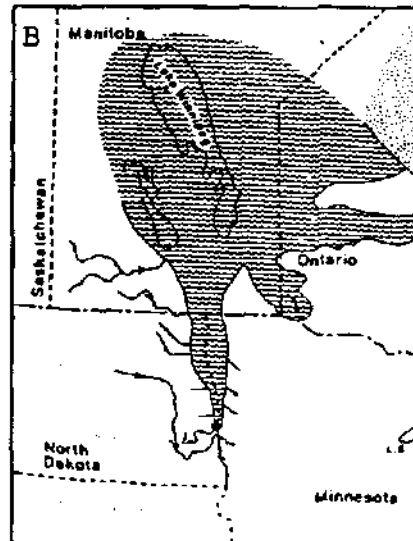
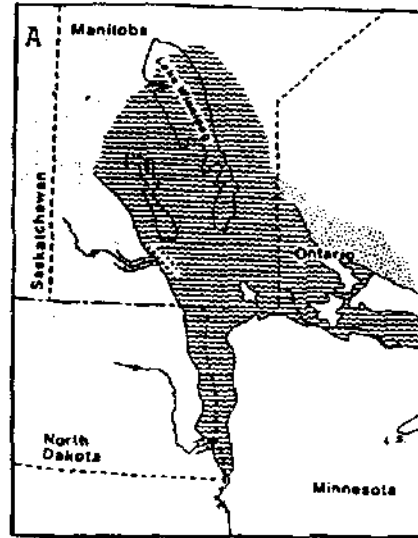
The Lockhart Phase consisted of the interval from 11,600 to 11,200 C<sup>14</sup> years B.P. This was marked by several small ice advances into the southern lake basin, but it was a time of general expansion of Lake Agassiz. The Campbell Beach complex formed during the Lockhart Phase, about 11,200 C<sup>14</sup> years B.P. (Figure 5a) Drainage of the Lake continued to be through the Minnesota River Valley (Fenton et al., 1983).









Figure 5:

A - The Campbell level of Glacial Lake Agassiz, approximately 11,500 years B.P. (Fenton et al., 1983, p 67).

B - Drainage of Lake Agassiz shifts from the Minnesota River to outlets leading to the Great Lakes (Fenton et al., 1983, p 67).

C - Lake Agassiz approximately 8,500 years B.P., after it had drained *from* present-day North Dakota (Fenton et al., 1983, p 70),



- Ice 
- Drainage 
- Underflow fan
  - Large 
  - Small 
- Entrenched river valley 
- Boundary of
  - Country 
  - Province or State 
- Lake Superior 

The Moorhead Phase (11,200 to 9,900 C<sup>14</sup> years B.P.) was marked by a steady drop in lake level as the ice retreated farther north and a succession of lower outlets to Lake Superior was exposed (Figure 5b). The portion of Lake Agassiz that covered present-day North Dakota is thought to have drained during this phase (Fenton et al., 1983).

A major readvance of the ice marked the beginning of the Emerson Phase, which lasted from approximately 9,900 to 9,500 C<sup>14</sup> years B.P. Lake Agassiz once again flooded parts of North Dakota as the low outlets to Lake Superior were closed off, raising the lake level back to that of the Campbell Beach. Drainage returned by way of the Minnesota River Valley (Fenton et al., 1983).

The final phase of Lake Agassiz was the Nipigon Phase, from about 9,500 to 8,500 C<sup>14</sup> years B.P. The ice retreated from the northern Lake Agassiz Basin one final time, reopening the low drainages to Lake Superior (Figure 5c). Lake Agassiz is believed to have drained from North Dakota by 9,000 C<sup>14</sup> years B.P., and by 8,500 C<sup>14</sup> years B.P. the lake had dried up (Fenton *et al.*, 1983), leaving Lake Winnipeg, Red Lake, and Lake of the Woods (among others) as remaining vestiges.

## ISOSTATIC REBOUND IN THE LAKE AGASSIZ BASIN

### Introduction

The most direct method of measuring rebound is from the strandlines left by glacial Lake Agassiz. The entire strandline presumably rebounded, with the northern end rebounding more because the ice was thicker in the north and had melted from that end later (Figure 6), the difference in elevation of the two ends of the strandline represents absolute minimum rebound. Strandlines formed in the early stages of Lake Agassiz will show the most rebound because less time passed between the retreat of the ice and formation of the strandline, thus there was less isostatic rebound prior to the strandline formation. Therefore, much of this study concentrates on the oldest well-developed Lake Agassiz strandline, the Herman.

Rebound can be determined from any strandline. The difference in rebound of two strandlines of known ages at the same distance from a common reference point gives the rebound rate.

It should be noted that rebound calculations based on elevation data represent absolute minimum rebound. The entire strandline experienced rebound (Figure 6) and as much as 73% of rebound was restrained, i.e., occurred as the ice was thinning (Figure 7) (Andrews, 1970, p 134). Also, before Lake Agassiz could have existed to form strandlines, the ice had to be gone completely. The Herman was not formed in the very earliest stages of Lake Agassiz; other beach remnants have

Figure 6 - The Herman strandline, showing greater uplift in the north than in the south (reference map revised from Johnston, 1946, p 2).



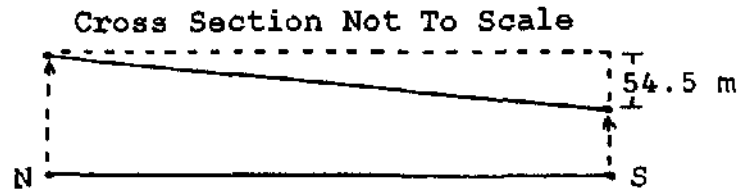
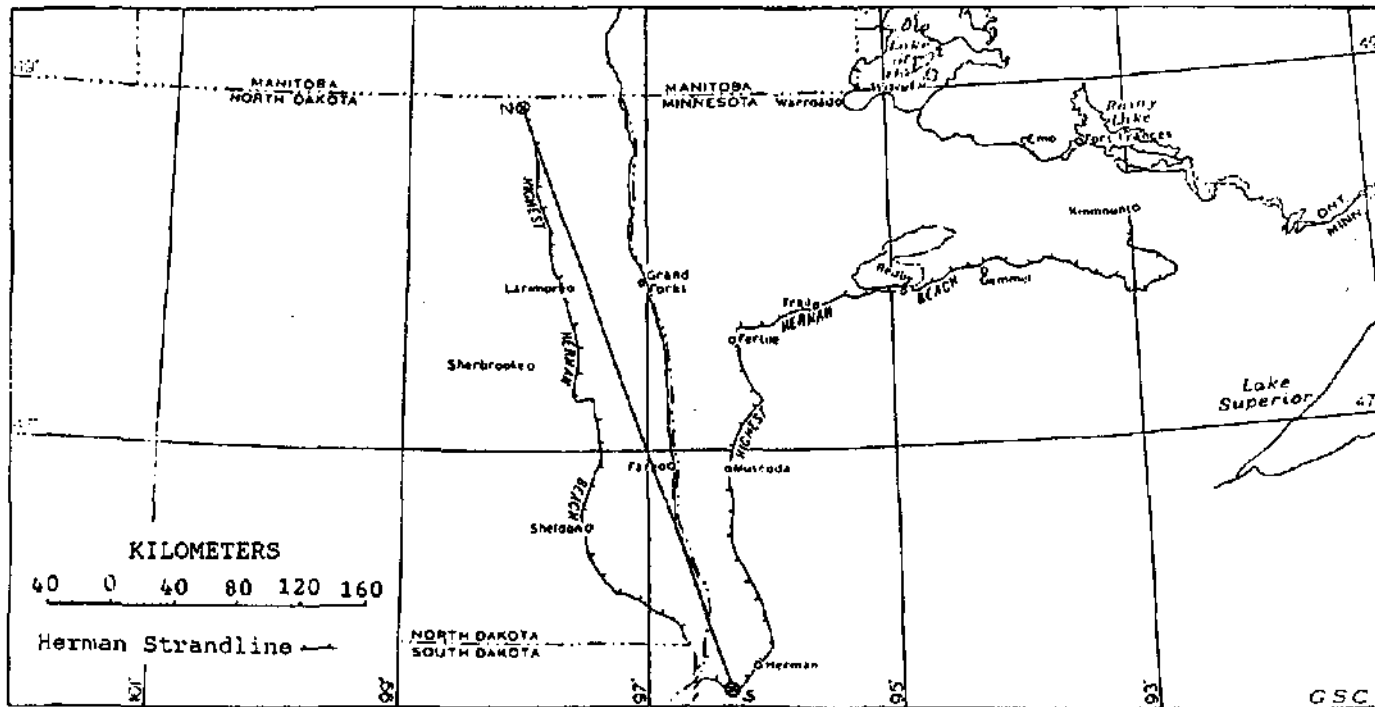
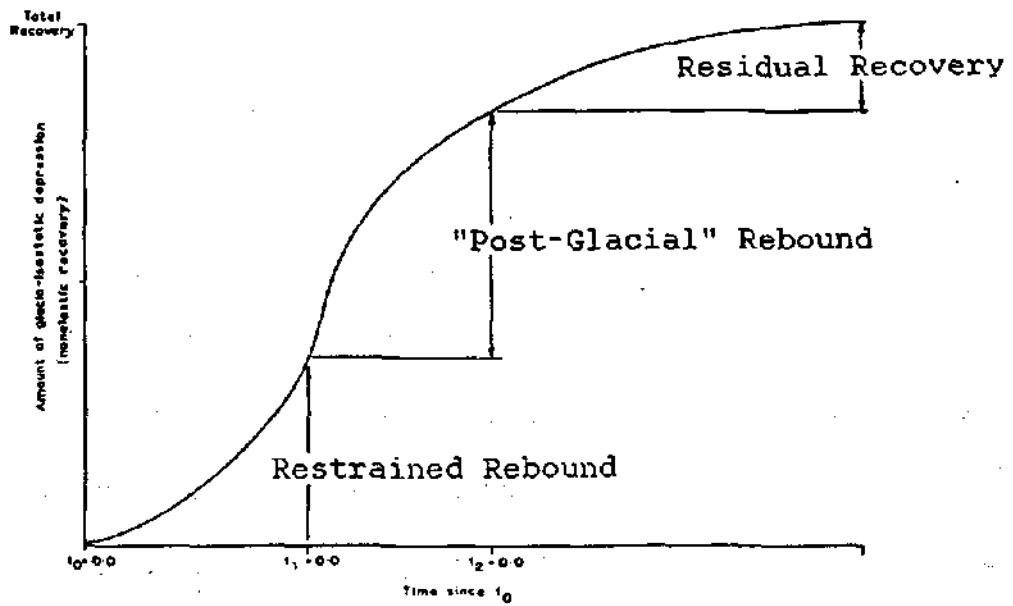


Figure 7 - Idealized isostatic rebound curve, showing restrained rebound (rebound before the ice completely melts), "post glacial" rebound (rebound after the ice melts), and residual rebound (minor rebound that remains until isostatic equilibrium is attained) (modified from Andrews, 1970, p 14).



been found as much as 30 meters above the Herman (Fenton et al., 1983, p 57).

Rebound began as the ice thinned and retreated, and the Herman strandline is not the oldest Lake Agassiz strandline. For these reasons, it is known that isostatic rebound began before the formation of the Herman strandline.

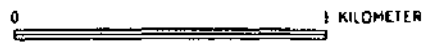
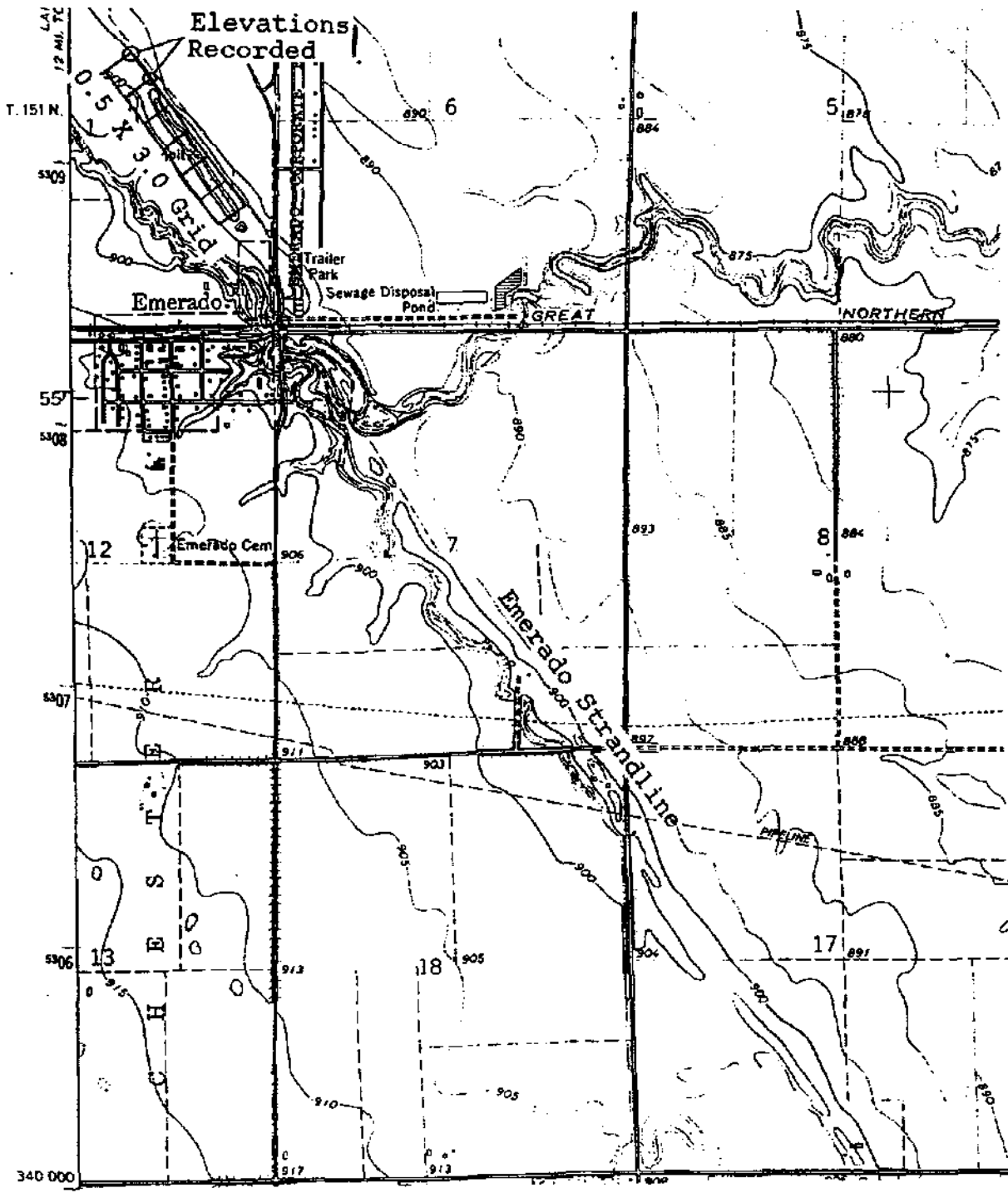
### Procedures

Each strandline was located using surficial geology maps from the North Dakota county ground water studies and the Geologic Map of North Dakota (Clayton, 1980). The individual strandlines are labeled on many of these, and can be located by township, range, and section. They can then be located on United States Geological Survey 7.5 minute topographic maps in order to determine elevation.

The elevation at a given point for each strandline was determined using a 0.5 by 3.0 centimeter grid, divided into 0.5-centimeter intervals. This grid was photocopied onto a transparency and placed over the strandline on the topographic map. The elevation at each intersection on the grid was recorded (Figure 8), and the average elevation calculated. If an intersection on the grid fell midway between two contour lines, the average of those contours was recorded. The error limit on elevation is +/- 1 meter.

The grid method was used in an attempt to minimize errors. Erosion would be affected by factors such as grain size, compaction, and vegetation cover. Consequentially, the elevation of a single point on the strandline at one place compared to the elevation of a single point at another may not represent the true

Figure 8 - Use of a 0.5 by 3.0 centimeter grid to find strandline elevation. The grid has been placed over the strandline on a USGS 7.5 minute topographic map. The circles on the grid show two of the points at which the elevation was read, both of which fall on the 900-foot contour line (Emerado Quadrangle, ND, 1967).



Location

difference in original elevation between those two points. By using a grid average, a single high or low point was not chosen on the strandline, thus reducing the chance of obtaining elevation values for the strandline that do not represent the original elevation difference between the measured points. For this reason, the grid average was used to provide a more accurate representation of the true difference in elevation than would be obtained by using single points.

### Calculation of Rebound

#### Elevation Diagrams

Elevation diagrams are a graphic means of portraying post-glacial rebound. The elevation data for each shoreline are plotted against the distance from a reference point on the Herman strandline (Figures 9, 10, and 11).

These diagrams show that the amount of rebound at any point along the beach can be found by subtracting the elevation at that point from the reference elevation, the lowest elevation on the beach. The values used to calculate crustal depression were from the Herman strandline. Values from the other diagrams will be used later in the discussion on residual rebound.

#### Calculations

The oldest well-developed strandline, the Herman, was measured to calculate absolute minimum rebound. The lowest point on the Herman strandline is 325.3 meters above sealevel. Its highest point in North Dakota, at the international border,

Figure 9 - The locations of data points used for the Herman strandline elevation diagram (Modified from Johnston, 1946, p 2).



Figure 10 - The locations of data points used for the Campbell strandline elevation diagram (Modified from Johnston, 1946, p 2).

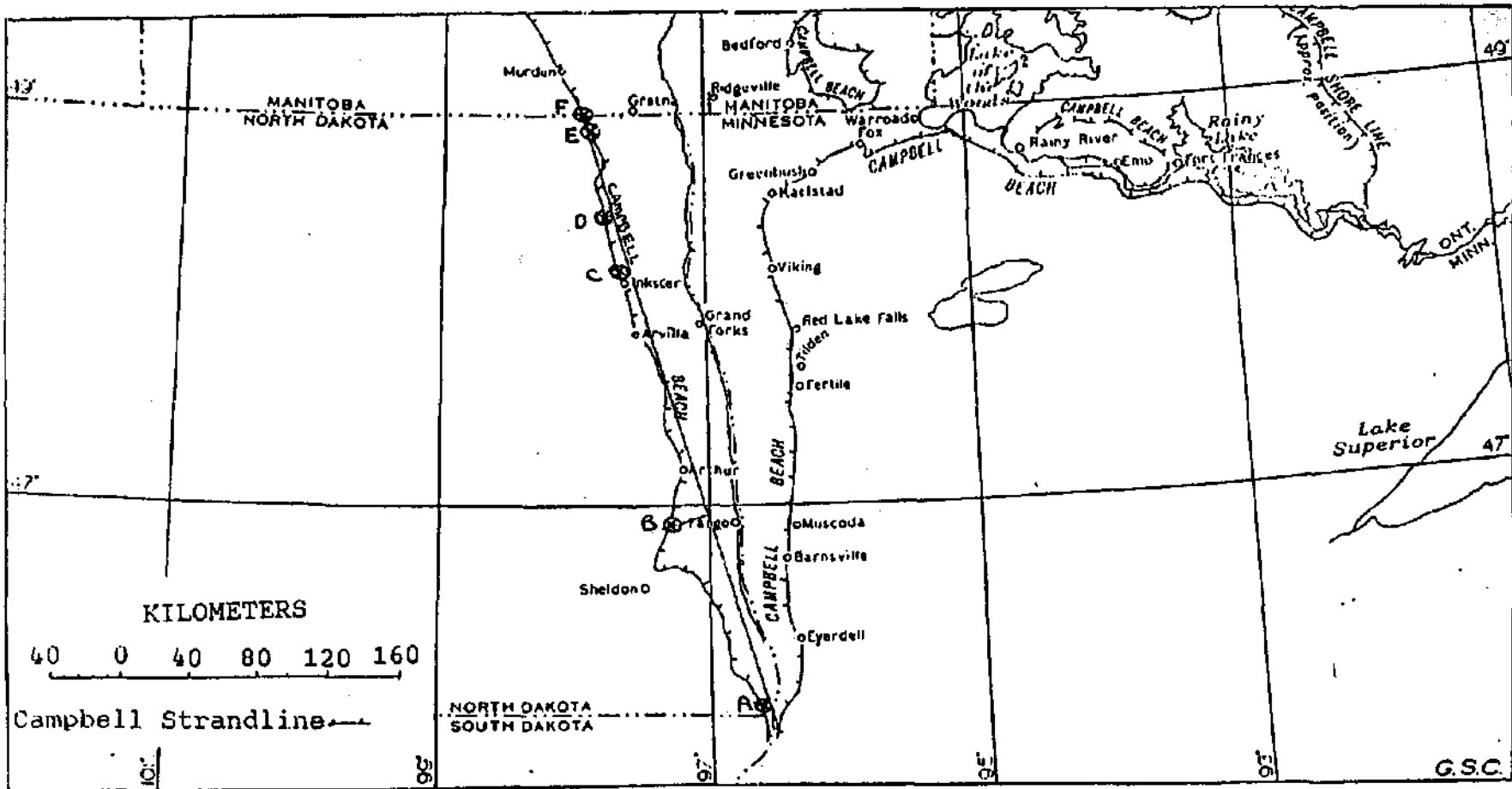
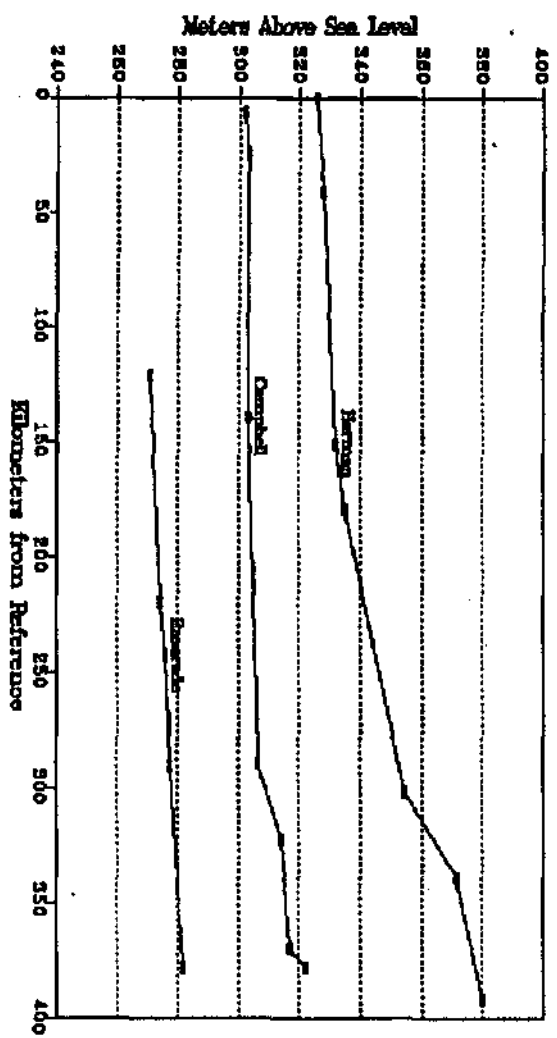


Figure 11 - Elevation diagrams for the Herman, Campbell, and Emerado strandlines, showing the difference in uplift between the northern (right side of diagram) and southern ends of the strandlines.



is 379.8 meters above sealevel (Table 1). This gives a minimum rebound value of 54.5 meters in this part of the Lake Agassiz basin.

Using Andrews (1970) value for restrained rebound, the maximum rebound was calculated to be 200 m. This amount represents the maximum depression in the basin due to the ice, if there was no rebound at the south end.

Table 1 - Lowest (south end) and highest (north end) elevations of selected Lake Agassiz strandlines. The difference is the absolute minimum rebound for the Lake Agassiz Basin in North Dakota.

<u>NAME</u>	<u>LOWEST POINT</u>	<u>INTERNATIONAL BORDER</u>	<u>REBOUND</u>
Herman	325 meters	380 meters	55 meters
Norcross*	320 meters	360 meters	40 meters
Tintah*	311 meters	341 meters	30 meters
Campbell	301 meters	322 meters	21 meters
Blanchard*	288 meters	300 meters	12 meters
Emerado	271 metres	282 meters	11 meters
Burnside*	250 meters	256 meters	6 meters

\* - modified from Bluemle, 1991, p 80

### Residual Rebound

#### Free-Air Gravity Anomalies

Depression of the crust by an ice sheet causes displacement of dense mantle material by viscous creep (Walcott, 1970, p 720). In a state of equilibrium, the free-air gravity anomaly is close to zero (Walcott, 1970, p 716). However, in the case of the Laurentide Ice Sheet, the ice retreated faster than recovery of the mantle, resulting in negative free-air anomalies (Walcott, 1970, p 719).

Walcott (1970, p 719) listed three main reasons why the existing anomalies are due to the Laurentide Ice Sheet: 1) the position, symmetry, and major axes of the anomalies correspond to those of the Laurentide Ice Sheet and its major centers; 2) the pattern of the anomalies corresponds to the pattern of deglaciation; and 3) studies of the tilt of marine strandlines define isobases that are parallel to the gravity contours (Figures 12a and 12b).

Note that on Figure 12a, the zero anomaly contour extends through the Great Lakes, then north to Lake Winnipeg and beyond. North Dakota is on the south side of this line where anomaly values are positive. This means that rebound in North Dakota probably is complete. Peltier (1989) used gravity data to produce a map of predicted rates of uplift in North America (Figure 13). Just as Walcott's anomaly maps suggest, Peltier concluded that rebound is complete in North Dakota. Rebound is not complete to the northeast, where free-air gravity anomalies are still negative.

### Strandline Evidence

Although the dating control on Lake Agassiz strandlines is not good, some data do exist (Table 2). These data support the findings of Peltier and the conclusions drawn from Walcott's free-air anomalies. From the time the Herman strandline was formed, up to the formation of the Campbell strandline, there was 34 m of uplift over 400 years. In contrast, in the time between the formation of the Emerado and Burnside strandlines, there was only 5 m of uplift over 400 years. The Herman

Figure 12 - The relationship between mean free-air gravity anomalies and ice thickness:

A - Mean free-air gravity anomalies for eastern Canada (Walcott, 1970, p 717).

B - Ice thickness of the Laurentide Ice Sheet. Note the similarity between the high negative anomalies and the thick sections of the ice sheet (Sugden, 1977, p 27).

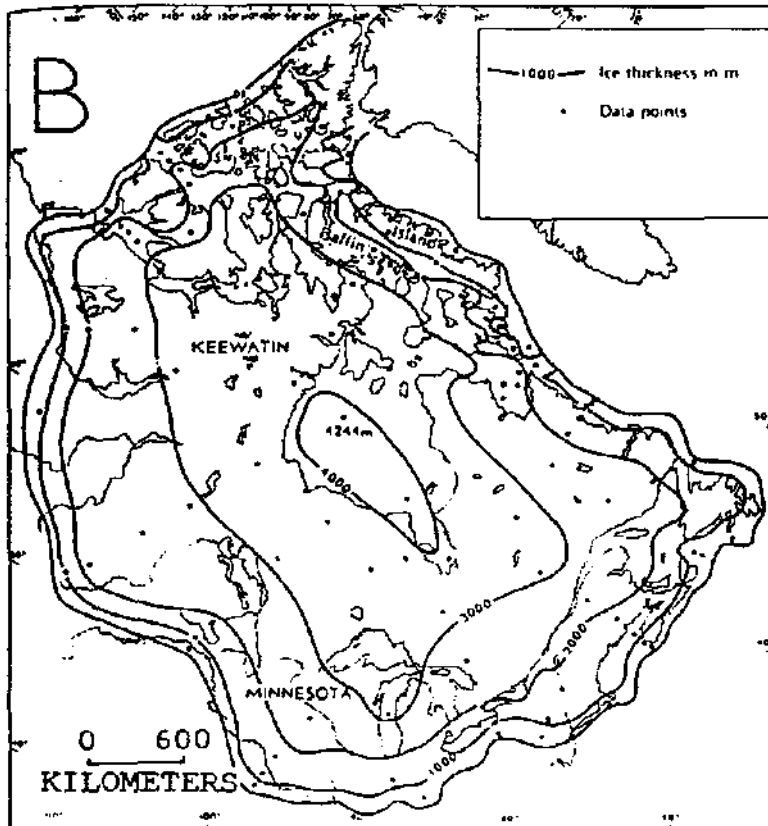
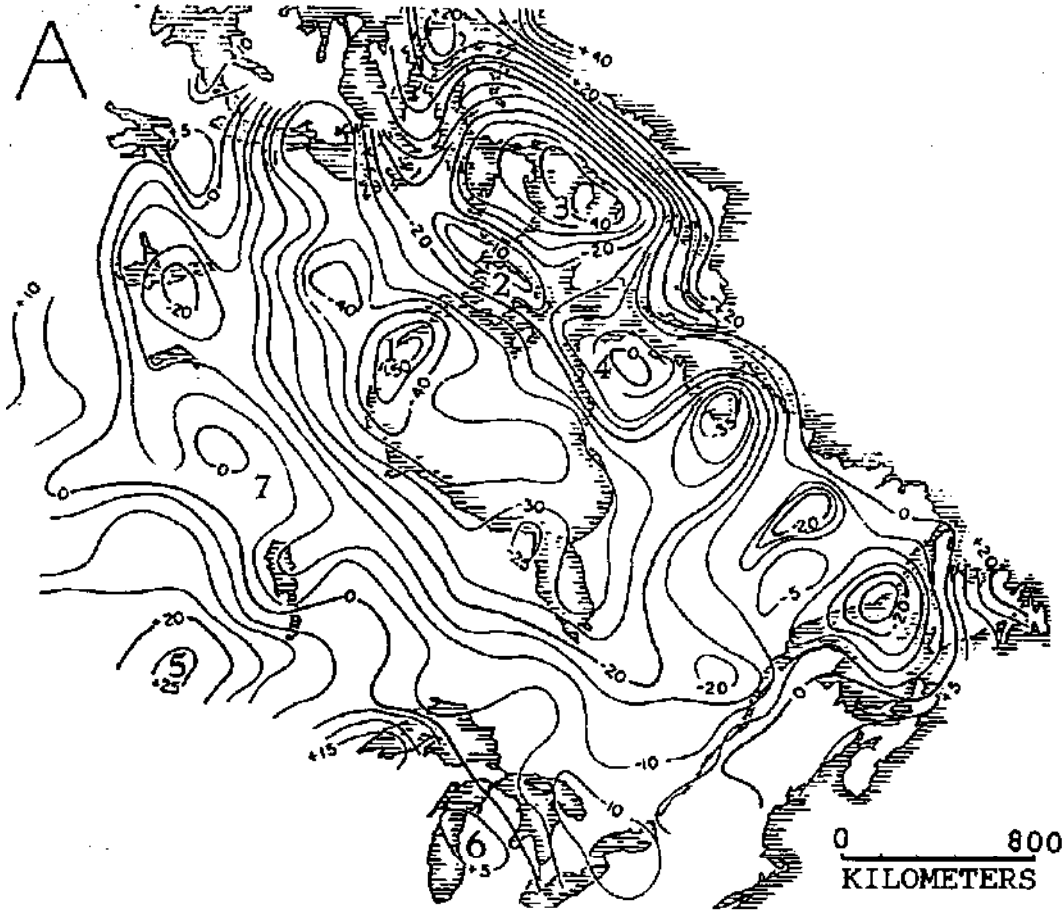
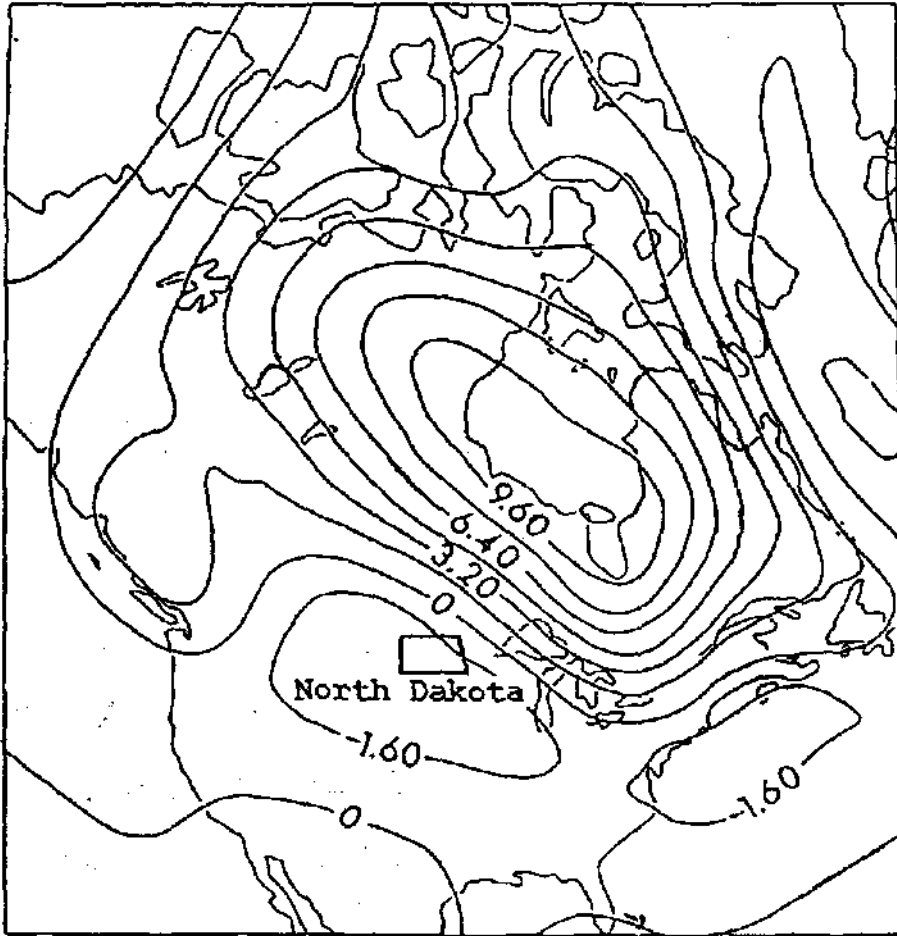




Figure 13 - Current uplift rates for North America in millimeters per year (Peltier, 1989, p 1447).



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Table 2 - Rates of uplift between strandlines, showing decreasing rate of uplift over time.

Strandlines	Uplift between Strandlines	Approximate Year Formed B.P.*	Rate of Difference (Years)	Uplift (m/centurv)
Herman to Campbell	34 m	11,600 11,200	400	9
Campbell to Emerado	10 m	11,200 10,900	300	3
Emerado to Burnside	5 m	10,900 10,500	400	1

\* - dates from Fenton et al., 1983

strandline was formed about 11,600 C<sup>14</sup> years B. P., whereas the Burnside strandline was formed approximately 10,500 C<sup>14</sup> years B. P. (Fenton et al., 1983). When rebound rates had decreased that much over a span of only 1,100 years, it seems that at present, approximately 10,500 years later, rebound should be complete. This can be checked if the viscosity of the mantle beneath the Lake Agassiz basin is known.

### Mantle Viscosity

The viscosity of the mantle is the major limiting factor in the recovery rate of depressed crust. Silver and Chan (1988) conducted experiments in the North American interior that suggest the viscosity of the upper mantle beneath the Canadian Shield is higher than average. Their findings are supported by Pinet et al. (1991), who found low heat flow values beneath the eastern Canadian Shield, indicating a high

viscosity value for the upper mantle. According to Gosnold (written communication, 1994), these findings suggest there may not be asthenosphere under the midcontinent of North America.

The amount of isostatic rebound that has occurred as a function of time is given by:

$$w = w_B e^{-t/T_M} \quad (1)$$

where  $w$  is the present amount of depression,  $w_m$  is the initial amount of depression,  $e$  is 2.71828,  $t$  is the amount of time elapsed since rebound began, and  $T_r$  is the mantle relaxation time (Turcotte and Schubert, 1982, p 247). Total elapsed time in the southern Lake Agassiz basin is 11,600 years (Fenton et al., 1983, p 64-65).

The mantle relaxation time is given by:

$$T_r = \frac{4\pi\eta}{g\rho_m\lambda^3} \quad (2)$$

where  $p_s$  is 3.14,  $\eta$  is the mantle viscosity,  $\rho_m$  is mantle density,  $g$  is gravitational acceleration, and  $\lambda$  is the wavelength of the ice sheet (Turcotte and Schubert, 1982, p 247). In the absence of an asthenosphere, mantle viscosity is  $1 \times 10^{21}$  Pa s (Turcotte and Schubert, 1982, p 248). Mantle density beneath the midcontinent of North America is  $3300 \text{ kg/m}^3$  (Braile, 1989, p 299), and the wavelength of the Laurentide Ice Sheet was about 3,000 km. Relaxation time for the upper mantle is calculated as 4024.893 years.

Absolute minimum depression in the Lake Agassiz basin was 54.5 m. In the absence of an asthenosphere, Equation 1 calculates that a minimum of 3.1 m of rebound would still need to occur in order for isostatic equilibrium to be achieved. At

maximum depression, calculated as 200 m, 11.2 m of rebound would remain. These values of residual rebound are underestimated, because if rebound is not complete the minimum and maximum values of depression calculated from the Herman strandline are too low. However, they do set minimum limits on the amount of rebound that may remain.

The gravity anomaly that would be produced if there were 11.2 m of residual rebound can be calculated by:

$$g. = 0.04193\rho_m h, \quad (3)$$

where  $g.$  is the gravity anomaly produced by the missing mass and  $h$  is the amount of residual rebound (Robinson and Coruh, 1988, p 260). Mantle density was used in the calculation instead of crustal density because the missing mass will eventually be made up by mantle material.

The viscosity of the mantle when the asthenosphere is present is  $4 \times 10^{19}$  Pa s. Equation 2 calculates relaxation time for the asthenosphere as 160.996 years, and Equation 1 calculates that isostatic rebound in the southern Lake Agassiz basin would be complete if the asthenosphere is present. The free-air gravity anomalies suggest that isostatic rebound is complete in the southern Lake Agassiz basin. However, it is possible that these results are misleading. Walcott's (1970) map of free-air gravity anomalies has a contour interval of 5 mgals (Figure 12a), but the calculated maximum residual rebound, 11.2 m, would cause an anomaly of only 15 mgals. Therefore, the anomaly caused by the residual rebound is less than the margin of error on Walcott's (1970) map. Sharma (1984) also points out that the use of free-air gravity anomalies

to indicate residual rebound has been questioned by several researchers. Therefore, the free-air gravity anomaly map is not conclusive.

The viscosity of the mantle beneath the Lake Agassiz basin was calculated using the Lake Agassiz strandlines. Equation 1 was used to estimate the amount of depression over time, assuming an experimental mantle viscosity. From this, an experimental amount of uplift between the formation of the four beaches listed in Table 2 was found. This was then compared to the observed uplift. The experimental viscosity that produced uplift values closest to the observed uplift values is the viscosity of the mantle beneath the Lake Agassiz basin.

Mantle viscosity is calculated at between  $9.5 \times 10^{19}$  and  $9.6 \times 10^{19}$  Pa s for the Lake Agassiz basin (Table 3). Given this viscosity, isostatic equilibrium would have been achieved 7,900 years B.P., 3,700 years after deglaciation. This indicates that rebound in the Lake Agassiz basin is probably complete.

### Discussion

It can be concluded that the Lake Agassiz Basin was depressed at least 54.5 m by the Laurentide Ice Sheet. The basin was possibly depressed 200 m or more, because the Herman is not the highest Lake Agassiz strandline. Therefore, it is possible that more than 73% of the total rebound had occurred before formation of the Herman.

The gravity data and strandline observations indicate residual rebound ought to be complete in the southern Lake Agassiz basin. Calculations that take mantle

Table 3 - Observed rates of uplift compared to experimental rates of uplift for the Lake Agassiz strandlines.

MANTLE VISCOSITY =  $9.5 \times 10^{19}$  Pa s

Strandlines	Observed Uplift Between Strandlines	Experimental Uplift Between Strandlines
Herman to Campbell	34 m	34.9 m
Campbell to Emerado	10 m	10.5 m
Emerado to Burnside	5 m	5.8 m

MANTLE VISCOSITY =  $9.5 \times 10^{18}$  Pa s

Strandlines	Observed Uplift Between Strandlines	Experimental Uplift Between Strandlines
Herman to Campbell	34 m	34.7 m
Campbell to Emerado	10 m	10.5 m
Emerado to Burnside	5 m	5.9 m

viscosity into account indicate that some rebound may remain if the asthenosphere is absent. Neither of these has been conclusively proven over the other. Therefore, future calculations will consider both possibilities.

## OTHER FACTORS AFFECTING REBOUND

### Introduction

Meltwater drains down slope, away from most glaciers. However, the Lake Agassiz Basin represents a special situation. There, the drainage is to the north; as the ice retreated the meltwater was trapped against the ice, forming glacial Lake Agassiz. This lake introduced additional complications to calculating rebound. Not only is the timing between the melting of the ice and the formation of the Herman strandline an unknown, but other factors must be considered, such as the lake water and sediments deposited in the lake, and how much they may have altered the rate of rebound.

### Water

The restrained rebound may have been retarded as the relatively light ice was replaced by denser Lake Agassiz water. This can be checked by calculating mass balance:

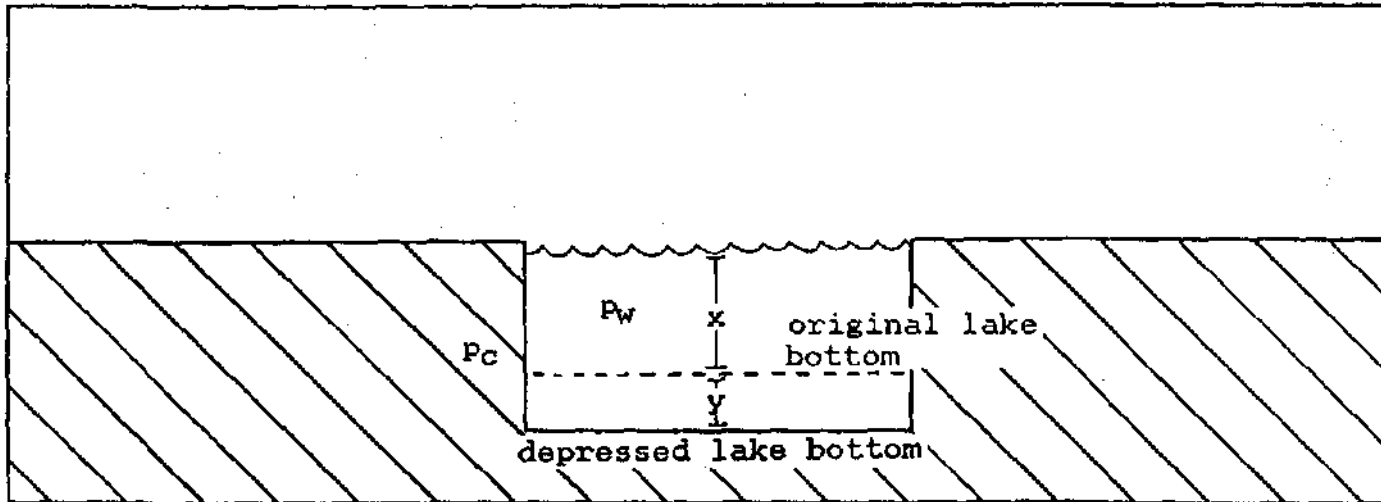
$$(C)(x + y)(p_w) = (p_c - p_w)(x + y)(C). \quad (4)$$

where C is a constant surface area of 1 m by 1 m,  $p_c$  is crustal density,  $p_w$  is the density of water, x is the height of the water above the depressed surface, y is the amount of crustal depression, and x + y is the total water depth (Figure 14). Units for Equation 4 are as follows:

$$(m^2)(m)(kg/m^3) = (kg/m^3)(m)(m^2).$$



Figure 14 - Depression of the crust caused by Lake Agassiz water, where  $p_c$  is crystal density,  $p_w$  is water density,  $y$  is the amount of depression, and  $x$  is the amount of water above the depression. The dashed line marks the original level of the lake bottom, prior to depression.



Cross cancellation of units leaves  $kg = kg$ , showing that the mass of the crust being displaced is equal to the mass that is displacing it.

Since  $C = C$ , it can be eliminated from the equation. In order to find the amount of crustal depression ( $y$ ) caused by a given amount of water, Equation 4 can be rearranged to:

$$y = ((x * \rho_c) / (\rho_c - \rho_w)) - x. \quad (5)$$

However,  $x$  and  $y$  are both unknowns. Therefore, substituting  $d^{\wedge}$  as a variable to represent the known water depth ( $x + y$ ), Equation 5 can be rearranged to:

$$x = d_w ((\rho_c - \rho_w) / \rho_c). \quad (6)$$

Depression ( $y$ ) can be found by subtracting  $x$  from  $d^{\wedge}$ , after solving for  $x$ .

The average depth of Lake Agassiz at Grand Forks was as much as 100 meters (Nordstog and Reid, 1984). With a density of  $1000 \text{ kg/m}^3$ , the mass of the lake water would have caused a crustal depression of 38 m at maximum average depth.

This depression is based on the assumption that the lake was at maximum depth long enough for the crust to reach isostatic equilibrium, but this assumption is not necessary. The lake formed as the ice retreated (Fenton et al., 1983, p 61), meaning the crust did not have time to rebound before the depressing effect of the lake began. The lake itself would not have caused the crust to depress any more than it already was, but the weight of the water would have retarded the rate of rebound.

## Sediment

The depression caused by lake sediments can be determined in the same way as that caused by water, substituting sediment thickness ( $d_j$  for water depth and the density of sediments ( $\rho_s$ ) for the density of water in Equation 6.

The Pierre Shale, which is composed mainly of silts and clays, has a density of about 2,100 to 2,200 kg/m<sup>3</sup> (Nichols et al., 1986, p 185). Much of the silt and clay deposited in Lake Agassiz was eroded from the Pierre Shale exposure along the Pembina Escarpment (Amdt, 1975, p 28). Because the silt and clay would have been water-saturated and uncompacted at the time of its deposition in Lake Agassiz, its density would have been less than that of the shale, probably a little less than 2,000 kg/m<sup>3</sup>. Stringers of sand and gravel, which have densities greater than 2,000 kg/m<sup>3</sup>, are also present within the lake sediments. Therefore, a density of 2,000 kg/m<sup>3</sup> is assumed for the lake sediments.

The sediments eventually accumulated to an average thickness of 46 meters in the Grand Forks area (Nordstog and Reid, 1984), which would cause an equilibrium depression of about 40 m.

Just as with the lake water, a depression of 40 m due to the sediments assumes adequate time to achieve isostatic equilibrium. However, because the sediments are still present in the Lake Agassiz basin, this assumption is unnecessary. The sediments did not cause any additional depression, rather, they continued to maintain, the 40 m of depression that had been caused by the lake water, and never allowed the final 40 m of rebound to occur (Figure 15).

Figure 15 - Steps in the depression of the Lake Agassiz Basin.

A - Pre-glacial position of the land surface.

B - Advance of the ice caused depression of the crust.

C - The crust began to rebound beneath Lake Agassiz, but rebound was retarded by the weight of the lake water and sediments.

D - The Lake Agassiz Basin today. The pre-glacial land surface is still depressed about 40 meters because of the Lake Agassiz sediments still present in the basin. However, the present land surface is a few (about 6) meters higher than the pre-glacial land surface because the lake sediments are not as dense as the crust. Therefore, the amount of depression caused by the sediments is a value less than the thickness of the sediments.

### Water and Sediment

The water level in Lake Agassiz did not remain constant, and sediments were deposited gradually over time. However, dating control on lake levels and sedimentation rates are not good enough to allow reliable correlation between the two. The maximum amount of depression caused by the Lake Agassiz water (38 m) and the sediments (40 m) is essentially the same. Therefore, for the purpose of this study, it has been assumed that the amount of depression caused by the combined effects of the water and sediments remained a constant 40 m. The contributions of various depths of water and various thicknesses of sediment to the total amount of depression are given in Tables 4 and 5, respectively.

### Discussion

The Lake Agassiz basin was occupied by water as the ice retreated, presumably right up against the ice margin (Bluemle, 1974, p 812). This means there was enough weight in the basin at all times to maintain about 40 m of depression. This limited total rebound.

Gradually, sediments were deposited, replacing the lake water. Sediments currently in the basin have enough mass to cause 40 m of depression, approximately the same amount as caused by the lake water. But, unlike the lake water, these sediments are still present, causing 40 m of depression. Therefore, between the lake water and the sediments, about 40 m of the original glacial depression never rebounded.

Table 4 - Crustal depression, resulting from various depths of Lake Agassiz, assuming a crustal density of  $2,670 \text{ kg/m}^3$  and a water density of  $1,000 \text{ kg/m}^3$ . Depression values have been rounded off to the nearest m.

<u>Depth of Water (m)</u>	<u>Crustal Depression (m)</u>
10	4
20	8
30	11
40	15
50	18
60	23
70	26
80	30
90	34
100	37

Table 5 - Crustal depression, resulting from various depths of Lake Agassiz sediments, assuming a crustal density of  $2.607 \text{ kg/m}^3$  and a sediment density of  $2.00 \text{ kg/m}^3$ .

5	4
10	8
15	11
20	15
25	18
30	23
35	26
40	30
45	34
50	37

Absolute minimum depression is actually represented by the tilt of the strandline plus the rebound that never took place, an amount equal to about 95 m

(Figure 15). Maximum depression, assuming a 73% restrained rebound, was approximately 350 m. This indicates that Lake Agassiz played a major role in affecting the rate and amount of rebound.

In the absence of the asthenosphere, and considering the effect of the Lake Agassiz sediments, minimum residual rebound would be between 5.3 and 19.6 m. A residual rebound amount of 19.6 m would cause a gravity anomaly of 2.7 mgals, still less than the contour interval of Walcott's (1970) free-air gravity anomaly map. The presence of the asthenosphere would indicate that rebound is complete, even given a maximum depression of 350 m. The mantle viscosity calculated using the strandlines indicates that rebound is complete.



## ICE THICKNESS IN THE LAKE AGASSIZ BASIN

### Introduction

Several methods have been proposed for the calculation of ice thickness along the marginal areas of the Laurentide Ice Sheet. One method is to calculate the thickness of ice necessary to cause the amount of depression calculated from the tilted Lake Agassiz strandlines. Another method, one that has received considerable attention and is cited frequently in the literature, was developed by Mathews (1974). This relies on a variable "A", a longitudinal ice slope factor, to determine thickness at a given distance from the edge of the ice tongue. A third method, which calculates thickness as a function of basal shear stress, was discussed by Beget (1987).

Each of these methods has advantages and disadvantages. Minimum depression can be measured directly from an uplift diagram, using field or topographic map data and making no assumptions. However, uncertainty in determining restrained rebound prior to the formation of any given strandline makes calculating a reliable maximum ice thickness difficult. Using a slope factor, Mathews (1974), has produced good approximations of the margins of current ice sheets. The present problem is finding the value of A for an ice sheet which no longer exists. Calculating ice thickness as a function of basal shear stress has the advantage of being fairly easy and straightforward, and it also lends itself to the determination of an ice profile. The biggest drawback is how to determine basal shear stress reliably for the time of glaciation.

The rebound amount from the Herman strandline plus lake sediments, Mathews' (1974) method, and the basal shear stress method, have been used to calculate ice thicknesses. These methods have been shown to be effective in calculating ice thickness along the margins of large ice sheets. The last two methods have also been used to calculate an ice sheet profile.

Other methods to calculate ice thickness, such as Nye's (1957) classic equation and Weertman's (1961) modifications to Nye's equation, cannot be used for the marginal areas of the Laurentide Ice Sheet; these methods were developed for glaciers that are frozen to or flow over a rigid substrate of high strength. Under these glaciers, basal yield strength is often on the order of 50 to 150 kPa (Beget, 1987, p 84). On the other hand, the sediments beneath the margins of the Laurentide Ice Sheet are believed to have been water-saturated, unconsolidated, and easily sheared, with a basal yield strength of only 1 - 22 kPa (Beget, 1987, pp 82, 84; Boulton and Jones, 1979, p 39). The gentle slopes of the marginal lobes of the Laurentide Ice Sheet are interpreted to have been the result of these low basal yield strength substrates (Boulton and Jones, 1979).

According to Fenton et al. (1983, p 58-59), the Late Wisconsinan ice advanced into this area approximately 20,000 years B.P. Fenton et al. (1983, p 60-61) concluded that the ice remained as far south as Des Moines, Iowa until about 14,000 years B.P., and covered the entire Lake Agassiz Basin until about 12,000 years B.P. It can be concluded that the Lake Agassiz Basin had glacial cover for approximately 8,000 years during the late Wisconsinan. For the purposes of this study, it has been

assumed that the ice was stable long enough for the crust to reach isostatic equilibrium.

### Calculation of Ice Thickness

#### Lake Agassiz Strandlines

The same principles that were used to calculate crustal depression due to water and sediments can also be used to calculate ice thickness. When the amount of crustal depression is known, Equation 4 can be rearranged in the following manner and solved for x:

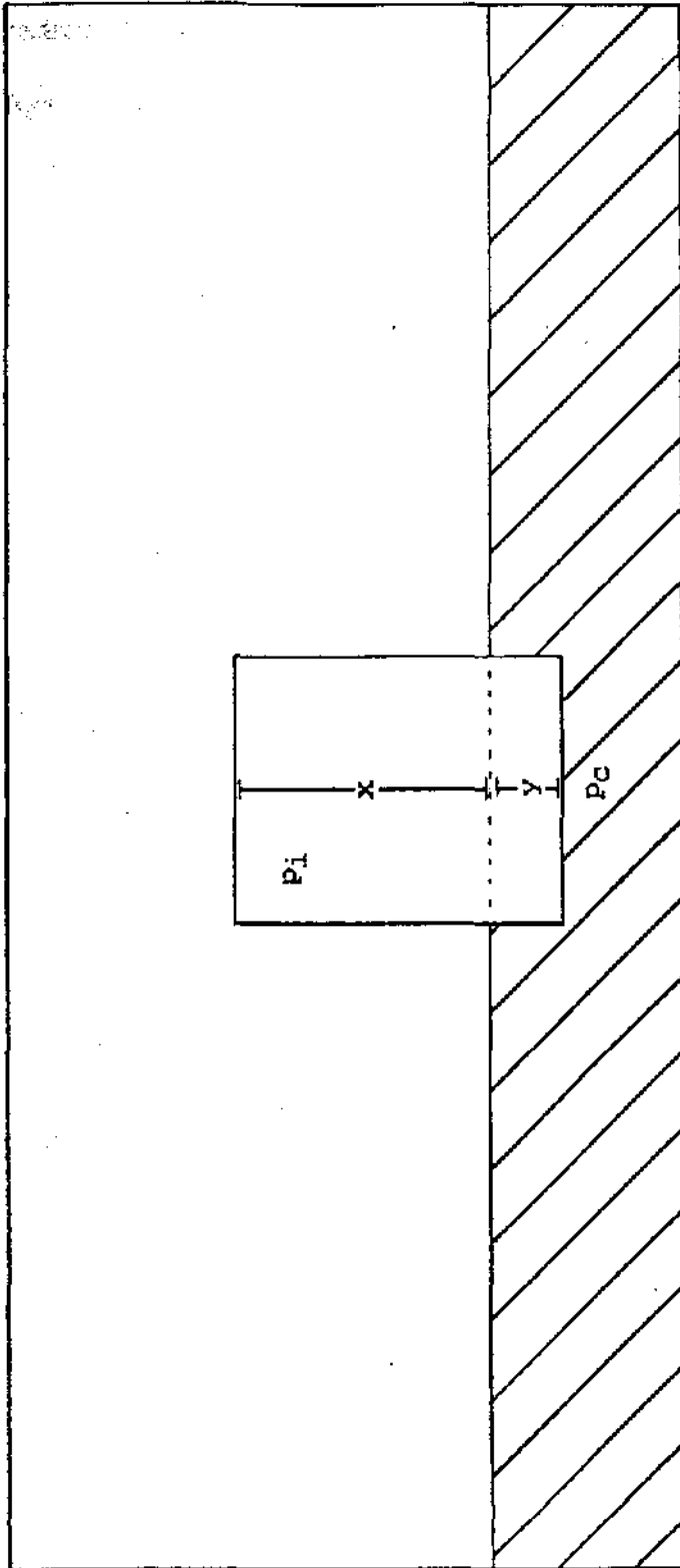
$$x = \frac{((p_c - p_j) / p_j) y}{((p_c - p_i) / p_i)} \quad (7)$$

where  $p_i$  is the density of ice (Figure 18). Ice thickness was then calculated by summing x and y (Figure 16).

The 95 m of minimum rebound (see p 18-19) reflects a minimum ice thickness of 280 m at Grand Forks, assuming an ice density of  $0.90 \text{ kg/m}^3$  and a crustal density of  $2.67 \text{ kg/m}^3$ . As much as 73% of rebound was restrained, indicating an ice thickness of up to 1,040 m (i.e.,  $280 \text{ m} / (1.00 - 0.73)$ ), if the Herman was formed immediately following melting of the ice sheet and/or if the water and sediments of Lake Agassiz retarded rebound.

It is important to determine when the Herman strandline was formed in relation to deglaciation. If the Herman was not formed immediately following deglaciation, post-glacial rebound already began when the Herman was formed. This indicates that total rebound between the beginning of ice retreat and formation of the Herman was

Figure 16 - Depression of the crust by ice, where  $p_c$  is crustal density,  $p_4$  is ice density,  $y$  is the amount of depression, and  $x$  is the amount of ice above the depression. The dashed line marks the original level of the land, prior to depression.



the sum total of restrained rebound (up to 73%) plus any post-glacial rebound that had occurred. Lake Agassiz strandline remnants have been identified as much as 30 m above the Herman (Fenton et al, 1983, p 57), indicating that the Herman does not represent the very earliest stages of Lake Agassiz. Ice thickness may have exceeded 1,040 m because the Herman is not the oldest Lake Agassiz strandline and may not have formed immediately upon the melting of the ice sheet. However, it is unlikely that ice thicknesses exceeded 1,040 m due to the rebound retarding effect of Lake Agassiz water and sediments. In addition, Bluemle (1974) believes that the first Lake Agassiz strandlines may have formed on stagnant ice which surrounded the lake, in which case post-glacial rebound did not begin until about the time that the Herman strandline was occupied.

### Mathews' Method

#### Introduction

Mathews (1974) proposed that ice thickness can be determined for the marginal areas of the Laurentide Ice Sheet by:

$$H = Ax^2 \quad (8)$$

where H is ice thickness, x is the distance from the ice terminus, and A is a variable which is a function of the longitudinal slope of the glacier. The value of A can be determined by elevation measurements of moraines from topographic maps. This method has become the standard by which ice thickness in the marginal areas of the

Laurentide Ice Sheet is calculated, and has provided a basis for much of the work in this study.

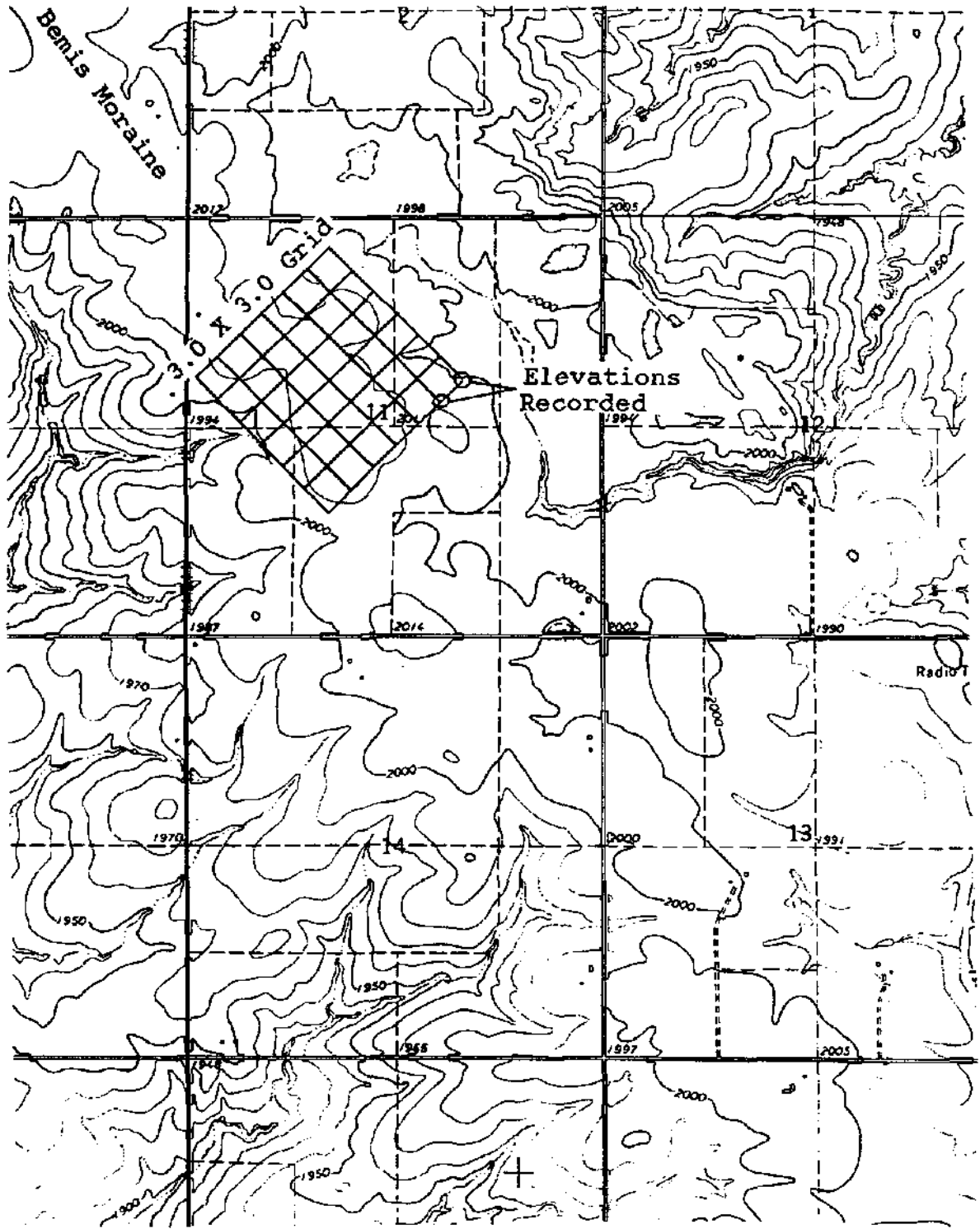
#### Procedures

The Bemis Moraine was located using the Geologic Map of Minnesota, Quaternary Geology (Hobbs and Goebel, 1982). Two locations on each side of the moraine were chosen, as well as a location at the terminus of the moraine. For the western location, the place where the Bemis crosses the Minnesota - South Dakota border was found, and the moraine traced back into South Dakota using USGS 7.5 minute topographic maps. Locations are specified by latitude and longitude on Hobbs and Goebel's (1982) map. This allows the same location to be found on United States Geological Survey 7.5 minute topographic maps and the moraine elevation to be determined.

The elevation at a given point for each side of the moraine was determined using a 3.0 by 3.0 centimeter grid, divided into 0.5 centimeter intervals, in the same way that elevation was determined for the strandlines (Figure 17). It is assumed that the same parabolic profile, from the ice crest near the base of the tongue to the lateral moraines, transverse to flow direction, is also applicable to the longitudinal slope which extends to the down-stream terminus (Mathews, 1974, p 40). Mathews used a trial value of A to create the two transverse profiles, thus finding an approximate crest elevation at their intersection (Figures 18 and 19). He then applied this approximation of "A", and the distance from the terminus to the transverse profiles, to plot a

Figure 17 - Use of a 3.0 by 3.0 centimeter grid to find moraine elevation. The grid has been placed over the moraine on a USGS 7.5 minute topographic map. The circles on the grid show two of the points at which the elevation was read as examples of how the grid was used. One of the points falls on the 2000-foot contour line, the other is on the 2010-foot contour (Toronto Quadrangle, SD, 1980).





Location

Figure 18 - Diagram illustrating the locations of the points used for elevation data on the Bemis Moraine. B and B<sup>1</sup> mark the western and eastern elevations taken from the Bemis moraine. C is at the former ice terminus. Line B - B<sup>1</sup> shows the location of the transverse profile; line C - C shows the longitudinal profile (Modified from Mathews, 1994, written communication).

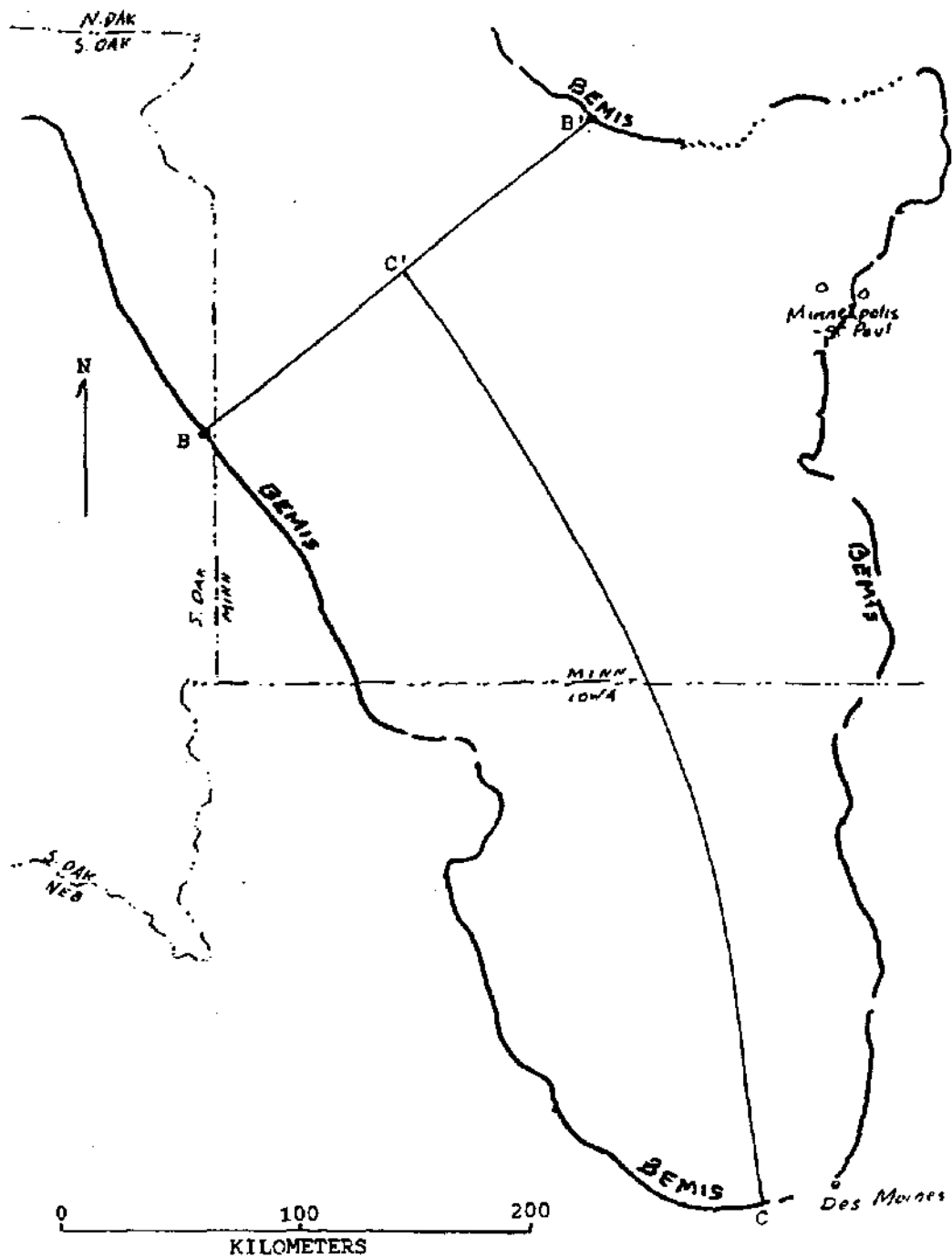


Figure 19 - Diagram illustrating procedure of defining the crest of the ice sheet (C). Two profiles are drawn perpendicular to flow direction. Where they intersect is the crest

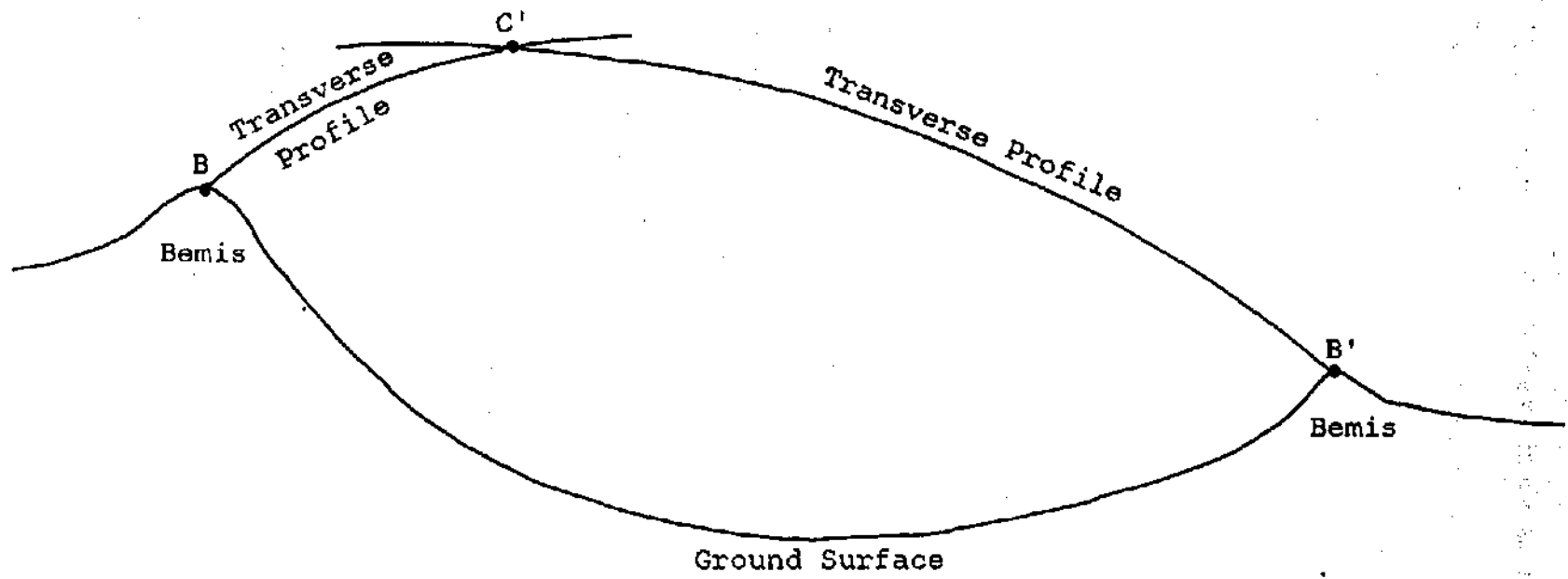


DIAGRAM NOT TO SCALE

longitudinal profile and determined the difference in elevation between the longitudinal profile and the crest of the transverse profile (Figure 20). Mathews stated that the first approximation is usually different from the value used for the transverse profiles, so a second iteration has to be performed, finding a new crest elevation, etc. (Mathews, written communication, 1994). The value of A that is accepted as the "true" value is the one that produces the closest match between the transverse crest elevation and the longitudinal crest elevation at the same location.

#### Determination of A

To determine A for this study, 6 trial values, ranging from 0.25 to 0.75, in 0.10 - step increments, were calculated. The closest fit was 0.45, so trial values of 0.44 and 0.46 were calculated, with 0.46 providing a closer fit than 0.45. An additional trial using 0.47 was also tried, but it did not fit as well as 0.46. The value for A used in this study is, therefore,  $0.46 m^m$  (Table 6). It is significant that this is the same value of A that Mathews obtained for the Des Moines Lobe, even though the points used on the eastern and western edges of the moraine are not the same as his.

#### Problems With A

Several assumptions must be made to arrive at an A value. The most critical is the assumption that the same A value applies to both the transverse and longitudinal profiles of the ice tongue. For example, Ackerly (1989) used the basal shear stress method to reconstruct transverse and longitudinal profiles for seven former glaciers in

Figure 20 - Diagram illustrating the use of the longitudinal profile in calculating "A". A longitudinal profile is drawn to the transverse profile, using the same value of "A", at distance D from the ice edge. The difference between the elevation of the longitudinal profile ( $C'_p$ ) at a distance D and the crest of the transverse profiles ( $C'_p$ ) is Z. Trial values of "A" are run until the smallest Z is found.

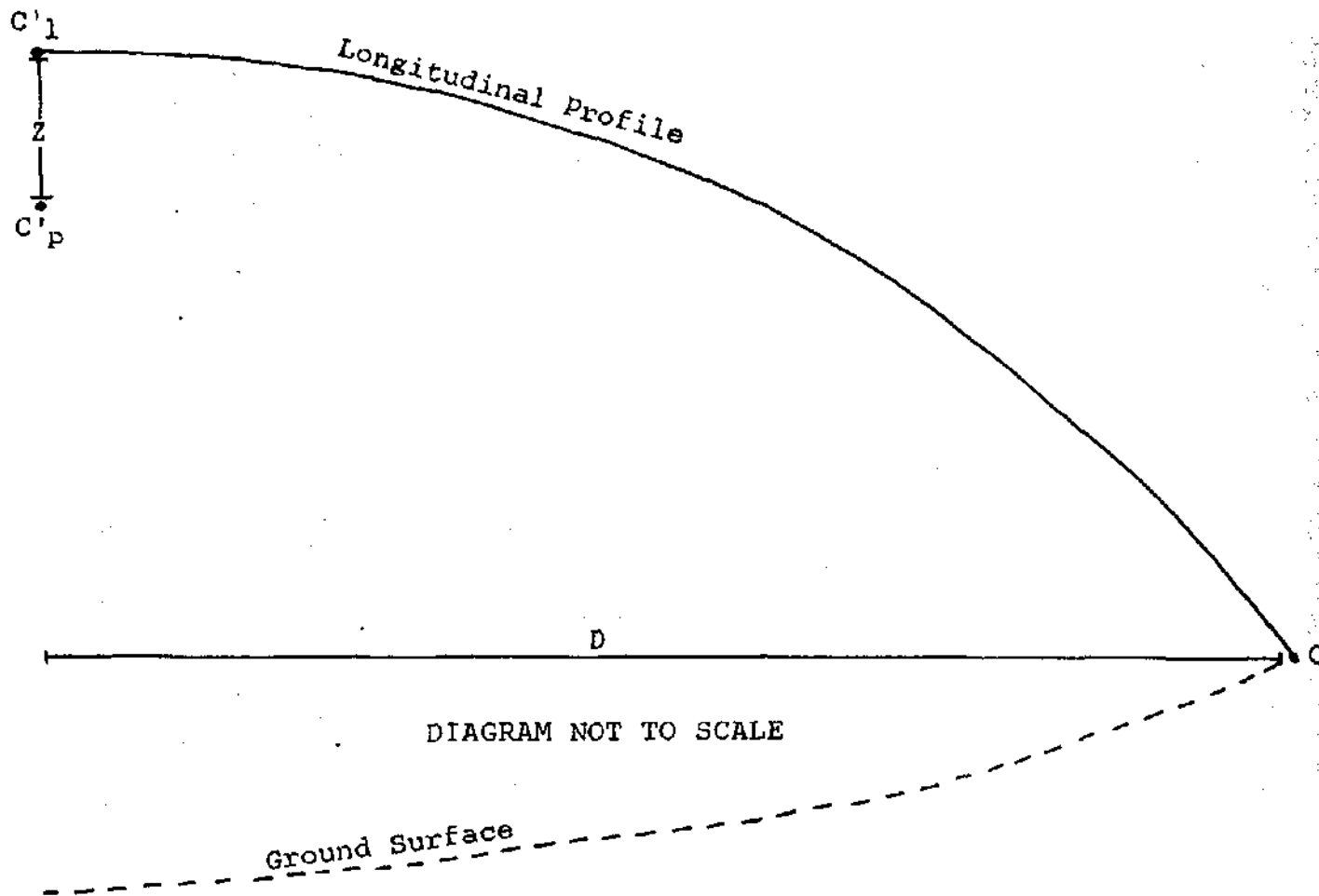


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Table 6 - The trial values used to determine A in this study. The accepted value of A is the one that causes the two profiles to come the closest to intersecting. Here, that is  $0.46\text{m}^{1/2}$ .

Trial Value of "A"	Crest of Transverse <sup>+</sup>	Crest of Longitudinal*	Difference
0.25	498	458	40 m
0.35	543	521	22 m
0.44	582.8	578.3	4.5 m
0.45	587.2	584.6	2.6 m
0.46"	591.7	590.9	0.8 m
0.47	596.2	597.3	1.1 m
0.55	630	647	17 m
0.65	675	711	36 m
0.75	710	774	64 m

+ - crest elevations are in meters above sealevel

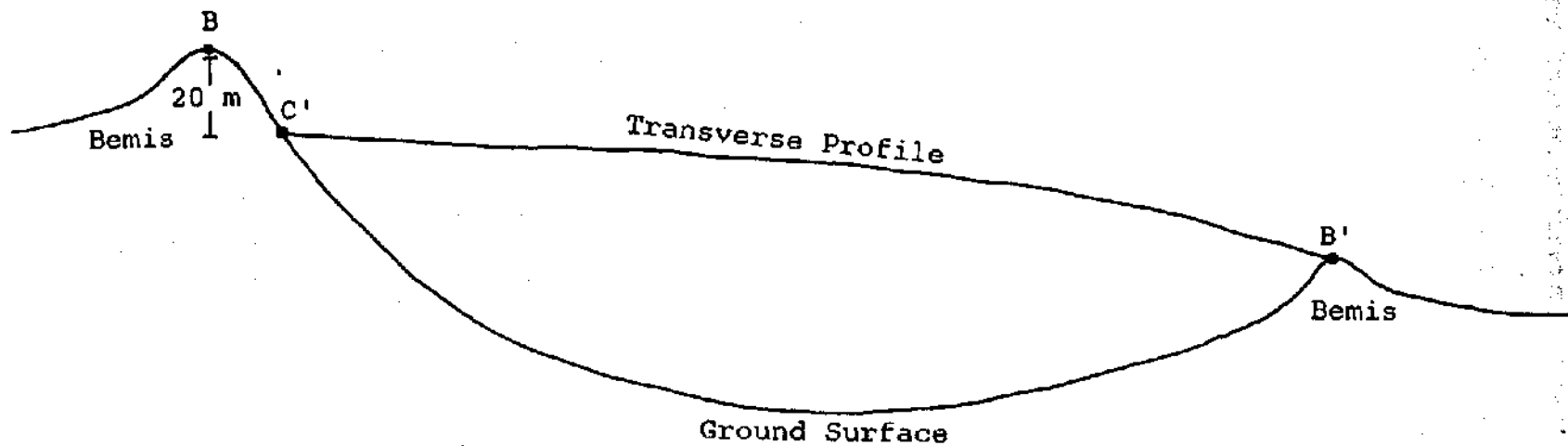
\* - best fit value of "A"

the highland areas of the northeastern United States. The transverse slope was not the same as the longitudinal slope on any of those glaciers.

Another problem is that the value of A makes sense mathematically, but it does not necessarily make geologic sense. The eastern edge of the Bemis moraine has an elevation of 386 m; the western edge has an elevation of 612 m at the same distance from the terminus. With an A value of  $0.46\text{ m}^{1/2}$ , the crest of the transverse profile is at 592 meters (Table 6). Essentially, this means that the ice crest was right up against the western moraine, and about 20 m below the moraine crest (Figure 21). Although Mathews (1974) does not address this problem, it seems unlikely that an ice sheet would build a moraine 20 m above the edge of the ice because the height of a moraine is limited by the ice at its contact.

Figure 21 - Diagram showing how the value of "A", obtained for the Bemis Moraine, results in a transverse profile from B' that intersects the western side at a position below the moraine crest.

DIAGRAM NOT TO SCALE



Except for marginal thrusting, moraines are not much higher than the edge of the ice. Even in cases of marginal thrusting, moraines do not build up 20 m above an ice sheet (Reid, verbal communication, 1994). Therefore, the 20 m difference between the crest of the western moraine and the ice sheet poses a problem.

There are several possible reasons for the unresolved 20 m. First, the ice sheet was in all likelihood asymmetrical, with the ice thicker to the west. This is the case if the majority of precipitation came from the south or west during the Wisconsinan, as it does today. The Scandinavian Ice Sheet, for example, was asymmetrical during the Late Weichselian (equivalent to the Late Wisconsinan). The Scandinavian Ice Sheet was thicker in the west than in the east (Nesje et al., 1988, p 160), because as moisture-laden air moved over the ice sheet from the west it lost its moisture through precipitation, a process similar to the rain shadow effect of mountains (Vorren, 1979, p 30-31). The additional ice on the west side of the ice lobe would have caused more depression and, consequently; the west would have rebounded more after the ice melted. The second reason that may explain the asymmetry of the ice lobe is that the bedrock is different beneath the two sides of the Bemis moraine. The western side is underlain mainly by shales, with the Pierre Shale as the main bedrock unit (Matsch et al., 1972, p 6-8). In contrast, the eastern side of the moraine is largely underlain by sandstones and granite (Winter and Norvitch, 1972, p 8-9). Therefore, the differences in bedrock lithology may have contributed to the noted 20 m difference. Finally, it is possible that what has been identified as the Bemis moraine is really two or more moraines that have been correlated wrongly. This explains the problem with applying

Mathews\* method to this "moraine". However, this possibility is considered unlikely as the area that includes the Bemis moraine has one of the best radiocarbon databases available for the Laurentide Ice Sheet (Bryson et al., 1969, p 4).

It is difficult to determine reliable parameters of an ice sheet that no longer exists. To complicate things even more, there is no large-scale modern example of long, thin ice tongues discharging from a major ice cap, such as is presumed to have occurred at the marginal areas of the Laurentide Ice Sheet (Boulton and Jones, 1979, p 40). But, despite its problems, Mathews' method is respected and frequently cited, even in the most recent literature (Andrews, 1991; Beget, 1987; Beget, 1986; Clayton et al., 1985; Boulton and Jones, 1979; Sugden, 1977). Ice thickness calculations will be made using an A value of  $0.46 \text{ m}^2$ .

## Results

Now that the value of A has been assumed, ice thickness can be calculated for any point between the ice margin and the international border. Substituting 0.46 for A in Equation 8 gives:

$$H = 0.46x^{1/2}. \quad (9)$$

With this equation, ice thickness at Grand Forks was 390 m, only, and ice thickness at the international border was 424 m (Table 7).

Table 7 - Ice thickness calculated by Mathews' method, maximum basal shear stress (Basal Shear<sup>1</sup>), and minimum basal shear stress (Basal Shear<sup>2</sup>). Grand Forks and the international border are at about 725 and 850 kilometers from Des Moines, respectively.

Distance from Des Moines km	Ice Thickness		
	Mathews' (m)	Basal Shear <sup>1</sup> (m)	Basal Shear <sup>2</sup> (m)
0	0	0	0
50	103	239	76
100	145	338	108
150	178	414	132
200	206	479	152
250	230	535	170
300	252	586	186
350	272	633	201
400	291	676	215
450	309	718	228
500	325	757	240
550	341	794	252
600	356	829	263
650	371	863	274
700	385	895	284
750	398	927	294
800	411	957	304
850	424	986	313

#### Basal Shear Stress Method

The thickness and profile of a large ice sheet also can be calculated by:

$$H = (2t_b D / \rho_i g)^{1/2}, \quad (10)$$

where H is ice thickness,  $t_b$  is basal shear stress,  $\rho_i$  is the density of ice, g is gravitational acceleration, and D, the distance from the edge of the ice sheet (Beget, 1987, p 84). Andrews (1970, p 65) presented this same equation as  $H = 195 D^{1/2}$ , by solving the expression  $2t_b/\rho g$  for the conditions in arctic Canada.

Acceptable values of ice density and gravitational acceleration are easy to obtain. The problem is in obtaining a valid value for basal shear stress. Shear strength for modern tills can be determined by conducting laboratory tests, but factors such as post-depositional weathering, jointing, etc. will cause the measured shear strength to differ from the original shear strength (Beget, 1986, p 236).

Another way to determine basal shear stress is to use preserved flow tills that originated as basal till, because sediment rheology controls the morphology of a flow-till at its terminus (Beget, 1986, p 237). The yield strength of a flow till can be found by:

$$K = \rho_s g h / \pi (1 - r/90), \quad (11)$$

where  $K$  is the yield strength,  $\rho$  is the density of the flow till,  $g$  is gravitational acceleration,  $h$  is the thickness of the till at the terminus,  $\pi$  is the constant 3.14, and  $r$  is the surface slope of the flow till (Beget, 1986, p 237). It can be shown further that if pore-water pressure in the basal till is assumed to be equal to glaciostatic pressure, then  $\tau_b = K$ , where  $\tau_b$  is basal shear stress (Beget, 1986, p 237). For this method to work, the flow must have been formed from unaltered basal till. This seems to be somewhat contradictory; till that was sheared up from an ice sheet base and then flowed down the terminus would be altered. In fact, Lawson (1979, p 40) does not even recognize flow tills as being till, he prefers the term sediment flow. This is because he sees till only as the sediment deposited directly by a glacier. Lawson believes sediment flows have been altered beyond the point of being till.

Clayton et al. (1985) postulated that shear stress under the southwestern part of the Laurentide Ice Sheet ranged from 0.5 to 5 kPa. If these values are substituted for  $\tau$  in Equation 9, and an ice density of  $900 \text{ kg/m}^3$  is assumed, the equations:

$$H = 0.34 D^{1/2} \quad (\text{when } \tau = 0.5) \quad (12a)$$

$$\text{and } H = 1.07 D^w \quad (\text{when } \tau^* = 5.0) \quad (12b)$$

are obtained for the determination of minimum  $H$  and maximum ice thickness, respectively. The Des Moines Lobe of the Laurentide Ice Sheet reached its maximum position at Des Moines, Iowa. It is about 850 km from the international border to Des Moines, the distance to the maximum extent of the ice margin. Substituting 850,000 m for  $D$  in Equations 12a and 12b, values of 313 m and 986 m are obtained for minimum and maximum ice thickness, respectively (Table 7).

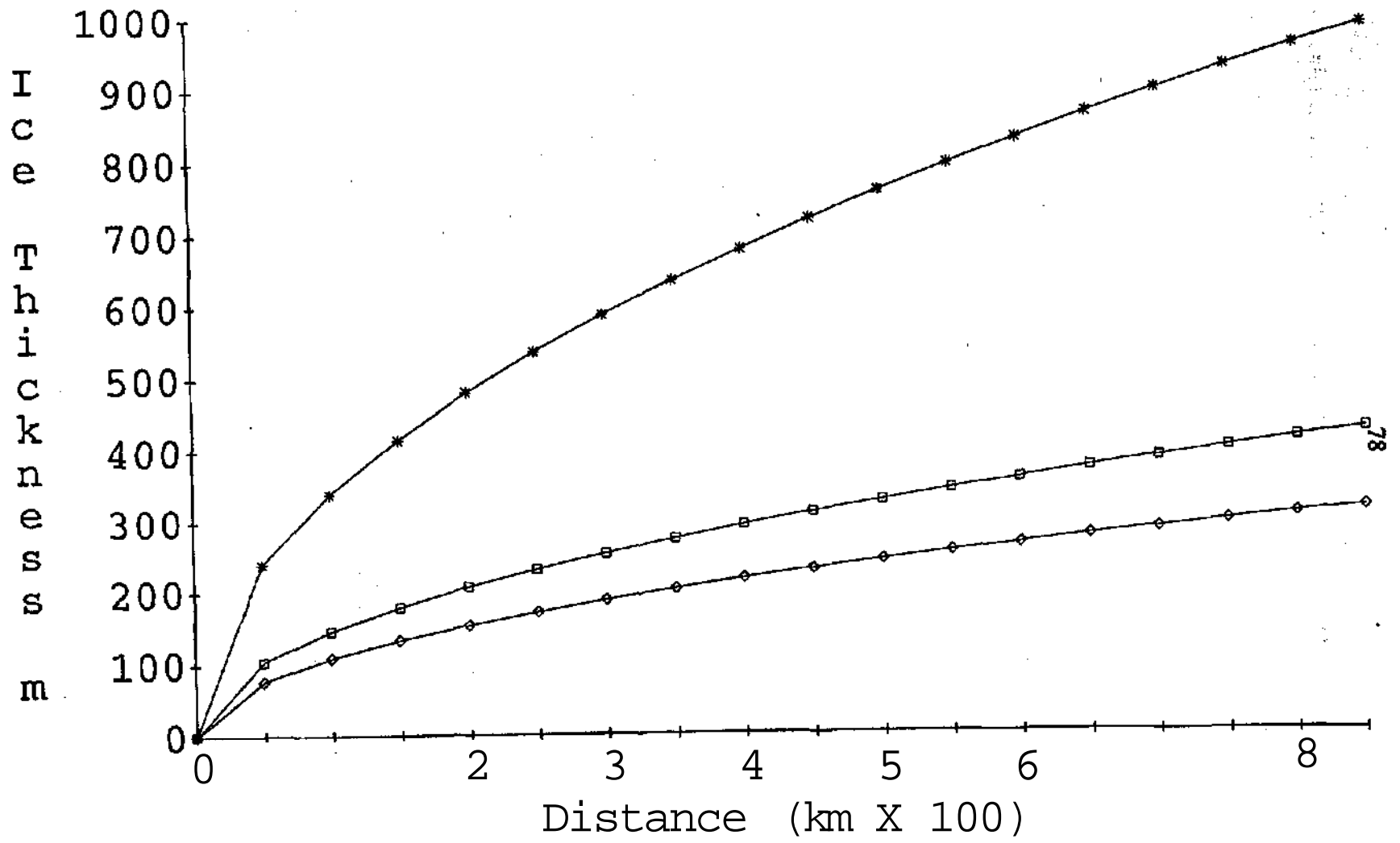
Studies in other marginal areas of the Laurentide Ice Sheet yielded basal shear stress values of 8 kPa and 7 kPa for Illinois and the Mackenzie Delta, respectively (Beget, 1986, p 238; Beget, 1987, p 84), which are close to the upper limits for the Lake Agassiz Basin of North Dakota, given by Clayton et al. (1985).

#### Calculated Ice Sheet Profiles

Ice sheet profiles can be created using the equations generated both by Mathews<sup>1</sup> method and by the basal shear stress method. Three different profiles have been generated, using 50 km intervals from Des Moines to the international border (Figure 22). These profiles represent ice thickness determined by Mathews' method



Figure 22 - Longitudinal ice sheet profiles from Des Moines, IA to the international border using Mathews' method with an "A" value of  $0.46 \text{ m}^{1/2}$ , the Maximum Basal Shear Stress method, and the Minimum Basal Shear Stress method.



•\*- Mathews      •\*- Max Bas      → Min Bas

and maximum and minimum ice thicknesses determined by basal shear stress, respectively.

### Expected Amount of Depression

Maximum depression of the Lake Agassiz Basin in North Dakota during the Wisconsinan can now be calculated. The maximum ice thickness from both Mathews' and the basal shear stress methods are used. Depression can be calculated using a rearranged form of Equation 10:

$$x = H((p_c - p_i) / p_c), \quad (13)$$

where ice thickness (H) is  $x + y$ . Depression (y) is then equal to  $H - x$ .

At the international border, Mathews' method and maximum basal shear stress result in ice thicknesses values of 424 m and 986 m, respectively. These values indicate maximum depressions of 140 m and 330 m, respectively (Table 8), both of which are greater than the minimum 95 m of depression indicated by the Herman strandline plus the Lake Agassiz sediments.

### Time Required to Achieve Isostatic Equilibrium

The amount of time required for the crust to reach isostatic equilibrium depends upon the viscosity of the upper mantle. When the asthenosphere is present, it takes less than 1,600 years for a depression of 54.5 m to rebound completely, and a depression of 350 m takes less than 2000 years to rebound completely. The rapid rebound indicated by the Lake Agassiz strandlines (Tables 2 and 3) suggests that an

Table 8 - Crustal depression resulting from various ice thicknesses in the Lake Agassiz Basin, assuming a crustal density of  $2,670 \text{ kg/m}^3$  and an ice density of  $900 \text{ kg/m}^3$ .

<u>Ice Thickness (m)</u>	<u>Crustal Depression (m)</u>
100	34
200	67
300	101
400	135
500	169
600	202
700	236
800	270
900	303
1000	337

asthenosphere does exist beneath the southern Lake Agassiz basin. However, due to the poor dating control of the strandlines, this cannot be certain.

Rebound rates are different if the asthenosphere is absent. A depression of 54.5 m would take over 37,600 years to rebound completely, and it would take over 44,800 years for a depression of 350 m to rebound completely. Table 9 compares some values of displacement with and without an asthenosphere.

Minimum and maximum depression in the Lake Agassiz basin have been calculated at 95 and 350 m, respectively. In order for the crust to reach isostatic equilibrium by the present given a 95 m depression, mantle viscosity beneath the Lake Agassiz basin cannot exceed  $2.86 \times 10^{20} \text{ Pa s}$ . If the crust was depressed 350 m, mantle viscosity cannot exceed  $2.53 \times 10^{20} \text{ Pa s}$  in order for isostatic equilibrium to be achieved by the present. Calculations made using the strandlines suggest that mantle viscosity beneath the Lake Agassiz basin does not exceed  $9.6 \times 10^{19} \text{ Pa s}$ .

Table 9 - Comparison of the amount of time required for a depression in the crust to complete rebound when the asthenosphere is present to the amount of time required to complete rebound when the asthenosphere is absent.

ELAPSED TIME	ASTHENOSPHERE PRESENT				ASTHENOSPHERE ABSENT			
	Displacement (m)				Displacement (m)			
<i>-Ml</i>	54.5	140	330	350	54.5	140	330	350
0	54.5	140	330	350	54.5	140	330	350
400	4.54	11.67	27.51	29.18	49.34	126.76	298.78	316.89
800	0.38	0.92	2.29	2.43	44.68	114.76	270.52	286.91
1200	0.03	0.08	0.19	0.20	40.45	103.91	244.92	259.77
1600	*0.00	0.01	0.02	0.02	36.62	94.08	221.75	235.19
2000	0.00	*0.00	*0.00	*0.00	33.16	85.18	200.78	212.94
10000	0.00	0.00	0.00	0.00	4.54	11.67	27.51	29.18
16000	0.00	0.00	0.00	0.00	1.02	2.63	6.20	6.57
20000	0.00	0.00	0.00	0.00	0.38	0.97	2.29	2.43
26000	0.00	0.00	0.00	0.00	0.09	0.22	0.52	0.55
30000	0.00	0.00	0.00	0.00	0.03	0.08	0.19	0.20
36000	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.05
37600	0.00	0.00	0.00	0.00	*0.00	0.01	0.03	0.03
40000	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.02
41600	0.00	0.00	0.00	0.00	0.00	*0.00	0.01	0.01
44800	0.00	0.00	0.00	0.00	0.00	0.00	*0.00	0.01
45200	0.00	0.00	0.00	0.00	0.00	0.00	0.00	*0.00

\* - approximate time that rebound is complete

#### Shape of the Depression at the Ice Edge

The weight of the ice causes an elastic upward bending of the lithosphere immediately beyond the margins of an ice sheet, known as a forebulge. Properties of the forebulge are controlled by the flexural parameter of the crust, which can be calculated by:

$$a = ((ET^3) / (3(1 - k^2) pg))^{1/4}, \quad (14)$$

where  $a$  is the flexural parameter,  $E$  is Young's Modulus,  $T$  is ice thickness at the center of the ice sheet,  $k$  is Poisson's ratio,  $\rho$  is the density of the underlying rock, and  $g$  is acceleration due to gravity (Walcott, 1970, p 721). Young's Modulus and Poisson's ratio have not been calculated for the rocks in the Lake Agassiz Basin (Gosnold, 1994, verbal communication). Therefore, data from Touloukain et al. (1981) were used to calculate an approximate value for these two variables (Table 10). Because the majority of the rock underlying the southern part of the Lake Agassiz Basin is granitic (Figures 2 and 3), the values for granite (Touloukain et al. 1981, p 135) were averaged. This average was then used to calculate the flexural parameter.

Young's Modulus and Poisson's ratio for the Lake Agassiz Basin will be assumed to be  $2.854 \times 10^{10}$  Pa and 0.15, respectively, the averages calculated from Touloukain et al. (1981). The density of the granitic bedrock is assumed to be  $2,670 \text{ kg/m}^3$ , a realistic average used by Robinson and Coruh (1988, p 286, 288).

It is important to calculate forebulge, because if the forebulge was large in relation to the amount of depression, it could affect the depression calculations, particularly those involving use of the strandlines.

The parameters of the forebulge can be estimated; for example, crest height can be estimated by:

$$H = T / 100 \quad (15)$$

where  $H$  is crest height and  $T$  is the thickness of the ice at the center of the sheet (Walcott, 1970, p 722). An estimate of how far the ground surface is depressed

Table 10- Young's Modulus and Poisson's ratio values for granite (from Touloukian et al., 1981, p 135).

Rock Type	Young's Modulus (GPa")	Poisson's Ratio
Granite	21.86	0.09
	34.82	0.19
	38.95	0.48
	16.06	0.03
	21.30	0.05
	31.65	0.15
	29.65	0.13
	32.40	0.14
	35.85	0.15
	26.89	0.14
	26.20	0.13
	26.89	0.12
	AVERAGE	28.54

below original equilibrium level (i.e., ground level prior to the ice advance) can be found by:

$$I = T/11.5 \quad (16)$$

where I is the amount of depression and T is the thickness of ice at the center of the sheet (Walcott, 1970, p 723). The distance from the crest of the forebulge to the ice edge is given by:

$$J = 1.9 a \quad (17)$$

where J is the distance from the crest to the ice edge and "a" is the flexural parameter (Walcott, 1970, p 723).

Because the marginal areas of the Laurentide Ice Sheet were considerably thinner than the central parts, the maximum thickness of the ice sheet at its center cannot provide reliable results for the marginal areas. For this reason, and because

this study involves only the southern Lake Agassiz basin (i.e., the portions south of the international border), the thickness of the ice sheet at the international border was used in all calculations involving ice *edge* conditions.

If the ice was 424 m thick, calculated from Mathews' method, the flexural parameter for the crust would have been 2,300 m, and the forebulge crest would have been 4.2 m at a distance of 4,400 m beyond the ice edge. Crustal depression at the ice edge would have been 37 m (Figure 23).

The maximum basal shear stress method results in an ice thickness of 1,042 m, corresponding to a flexural parameter value of 4,500 m. The forebulge crest would have been 10.4 m at a distance of 8,600 m beyond the ice edge. Crustal depression at the ice edge would have been 90 m (Figure 24).

The minimum basal shear stress method yields an ice thickness of 304 m. The calculated flexural parameter for this thickness is 1,800 m, with a forebulge crest of 3.1 m at a distance of 3,500 m beyond the ice edge. Crustal depression at the ice edge would have been 27 m (Figure 25).

The values for forebulge calculated here generally agree with values calculated by Newman et al. (1974, p 388), who determined that the forebulge caused by the Laurentide Ice Sheet was less than 20 m high at its crest. The values obtained by Newman et al. (1974) were based on sea level curves.



Figure 23 - Profile of the ice edge, using a thickness of 424 m, as determined by Mathews\* method (modified from Walcott, 1970, p 723).

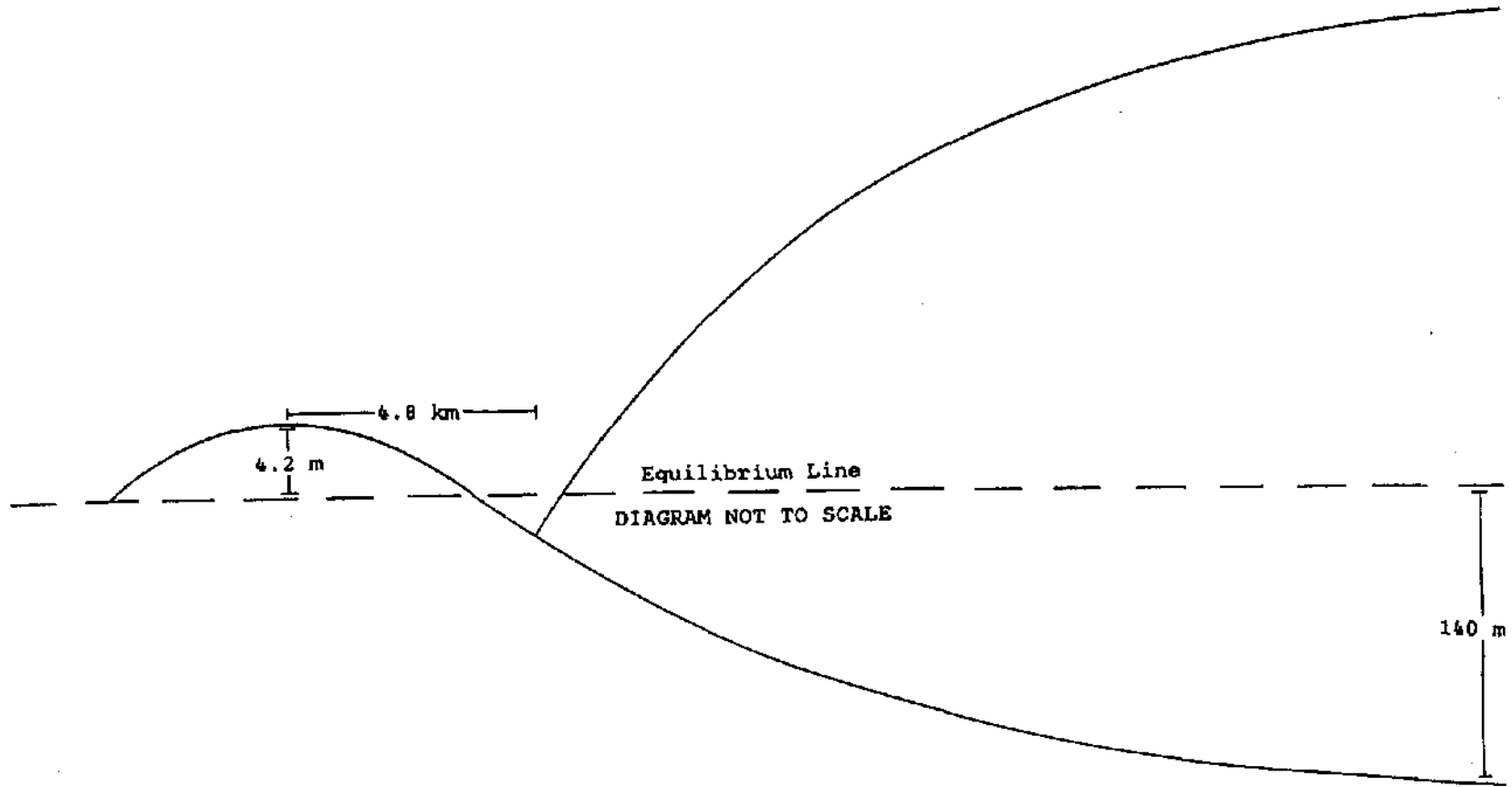


Figure 24 - Profile of the ice edge, using a thickness of 1042 m, as determined by the maximum basal shear stress method (modified from Walcott, 1970, p 723).

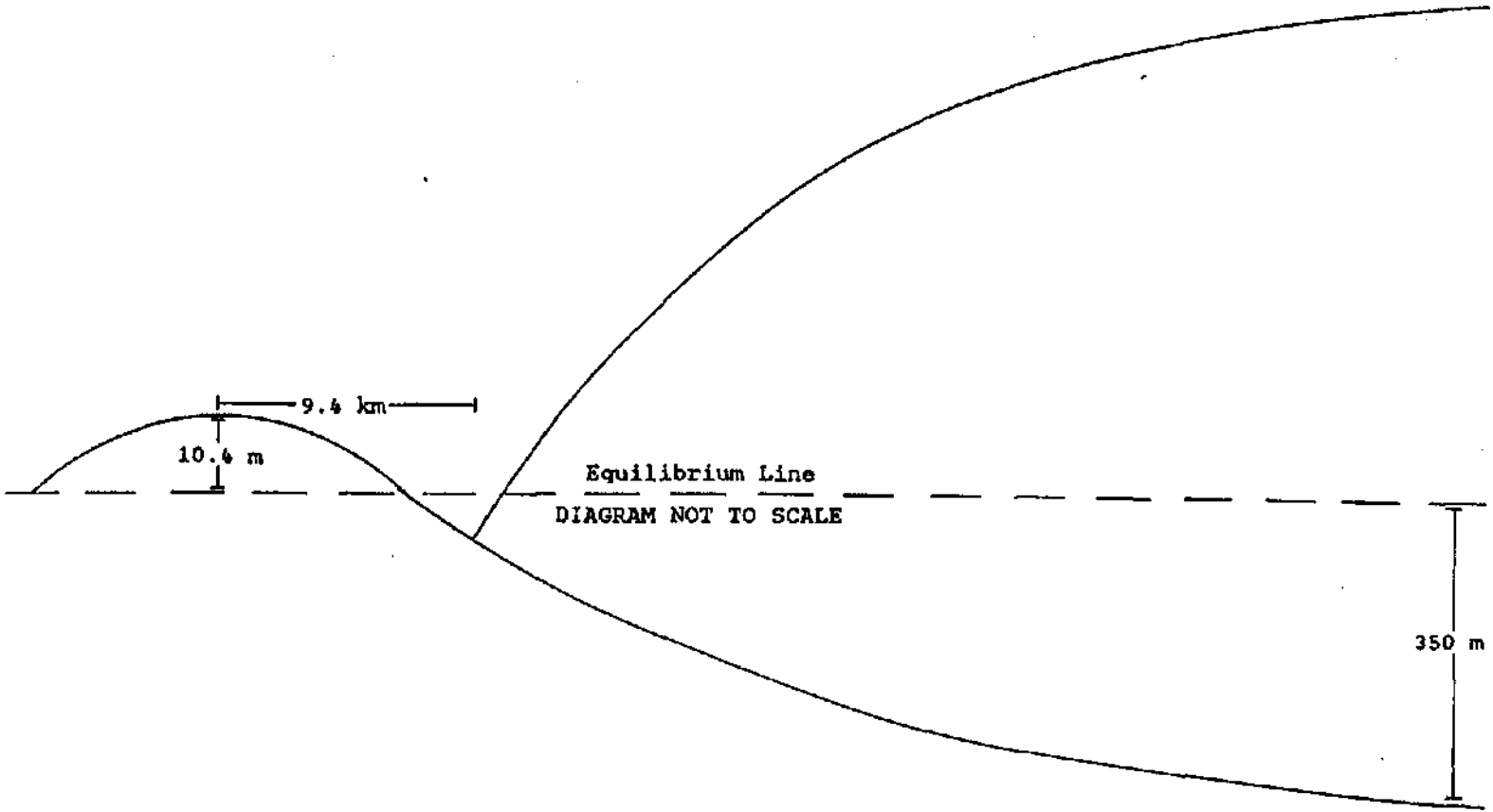
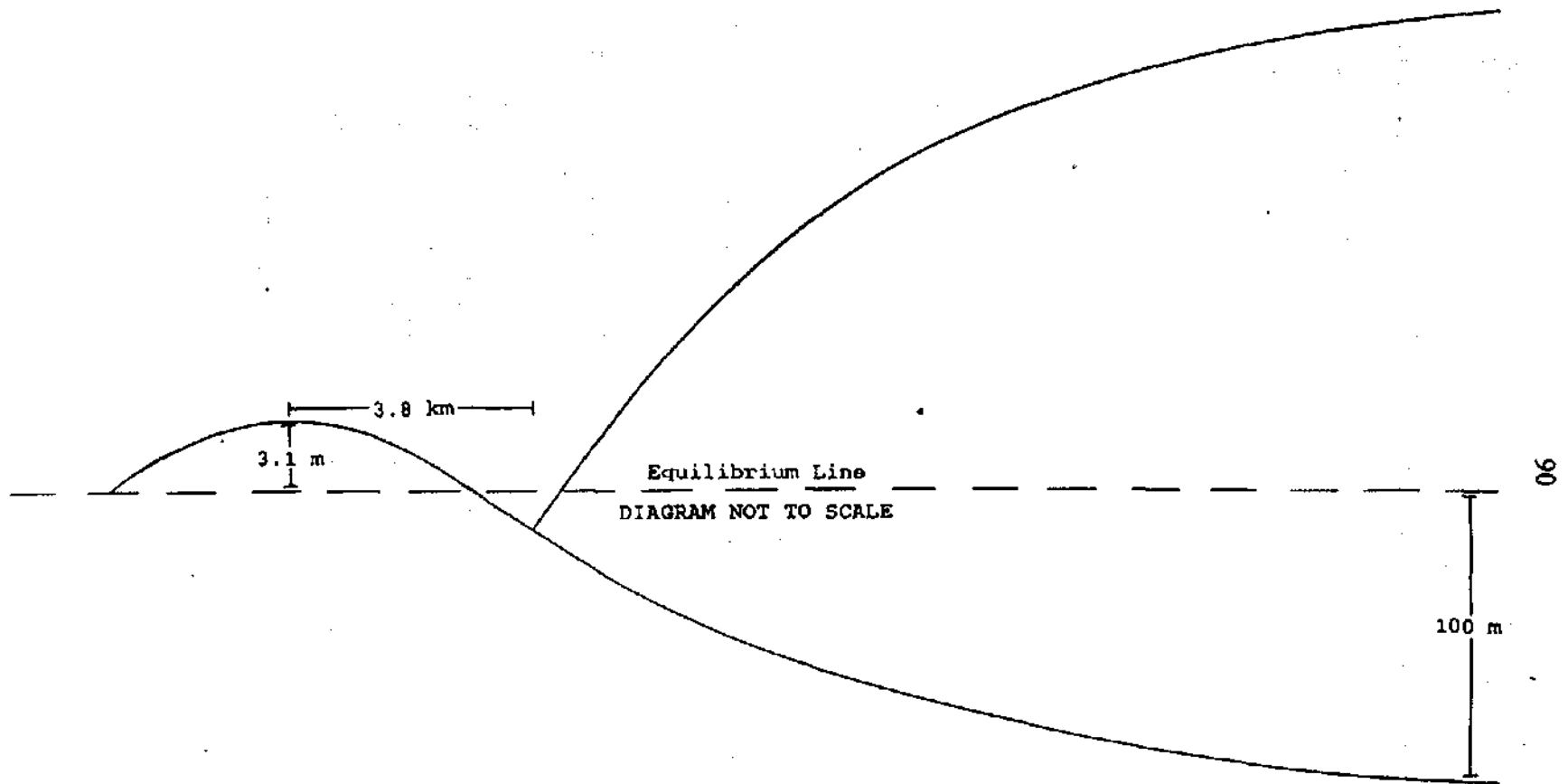


Figure 25 - Profile of the ice edge, using a thickness of 304 m, as determined by the minimum basal shear stress method (modified from Walcott, 1970, p 723).



### Discussion

Ice thicknesses calculated in this study, using methods developed for the marginal areas of large ice sheets, do not exceed 1,042 m at a distance of 850 km beyond the ice terminus. It is theorized that the reason the marginal ice was so thin is because the marginal portions of the Laurentide Ice Sheet flowed over soft, deformable sediments and poorly consolidated rocks (Boulton and Jones, 1979, p 40).

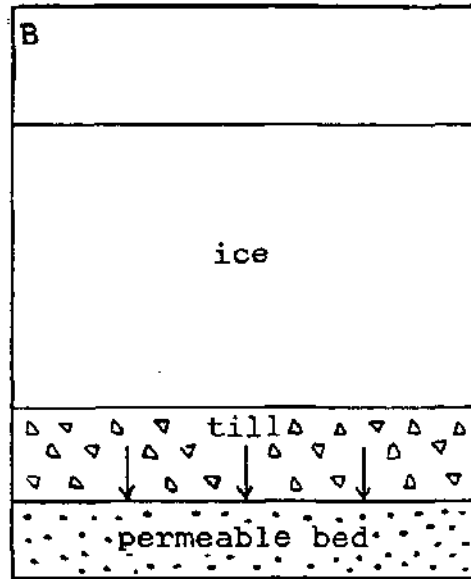
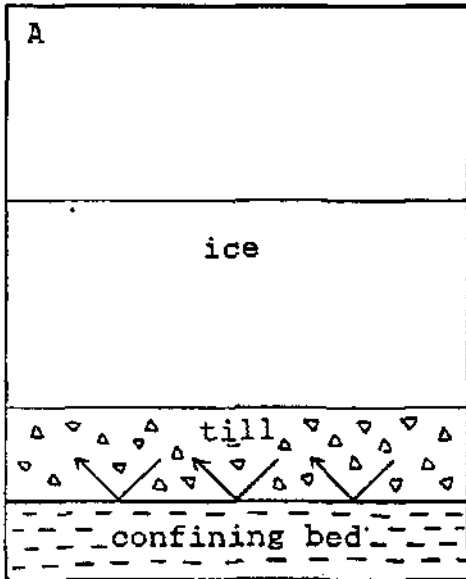
The main problem with this theory is that it cannot be tested with the large modern ice sheets on anything greater than a local scale. However, there are several lines of evidence that support the deforming-bed model. First, all low-profile glaciers that have been mapped, e.g., the southwest and northwest margins of the Laurentide Ice Sheet (Mathews, 1974, p 39, and Beget, 1987, p 82, respectively), and the Baltic Ice Stream between the English coast and the Dogger Bank (Boulton and Jones, 1979, p 36), occurred in low-relief sedimentary basins that have abundant unconsolidated sediments, limited bedrock obstructions, and are in the marginal areas of their respective ice sheet (Beget, 1986, p 238). Second, inherent in this theory is that there must be a confining bed beneath the deformable layer (Boulton and Jones, 1979, p 30, 38). Clayton et al. (1985, p 235) noted that whether the ice flowed over sandy till or clayey till made a difference. Glaciers have a normal, steep longitudinal profile over sandy till because subglacial water can drain through the till; pore-water pressure cannot build up beneath the glacier and a water-saturated, deformable bed cannot develop (Figure 26) (Clayton et al., 1985, p 237). Boulton and Jones (1979, p 39) also concluded that ice is thicker over strong, rocky substrates due to the lack

Figure 26 - Diagrams contrasting the behavior of subglacial pore-water in different situations:

A - A confining bed beneath the subglacial sediments will not allow basal melt water to escape, thus building up pore-water pressure and creating a deformable bed beneath the glacier.

B - A porous bed beneath the subglacial sediments allows basal melt water to escape, preventing pore-water pressure build-up. Thus no deformable bed is formed.

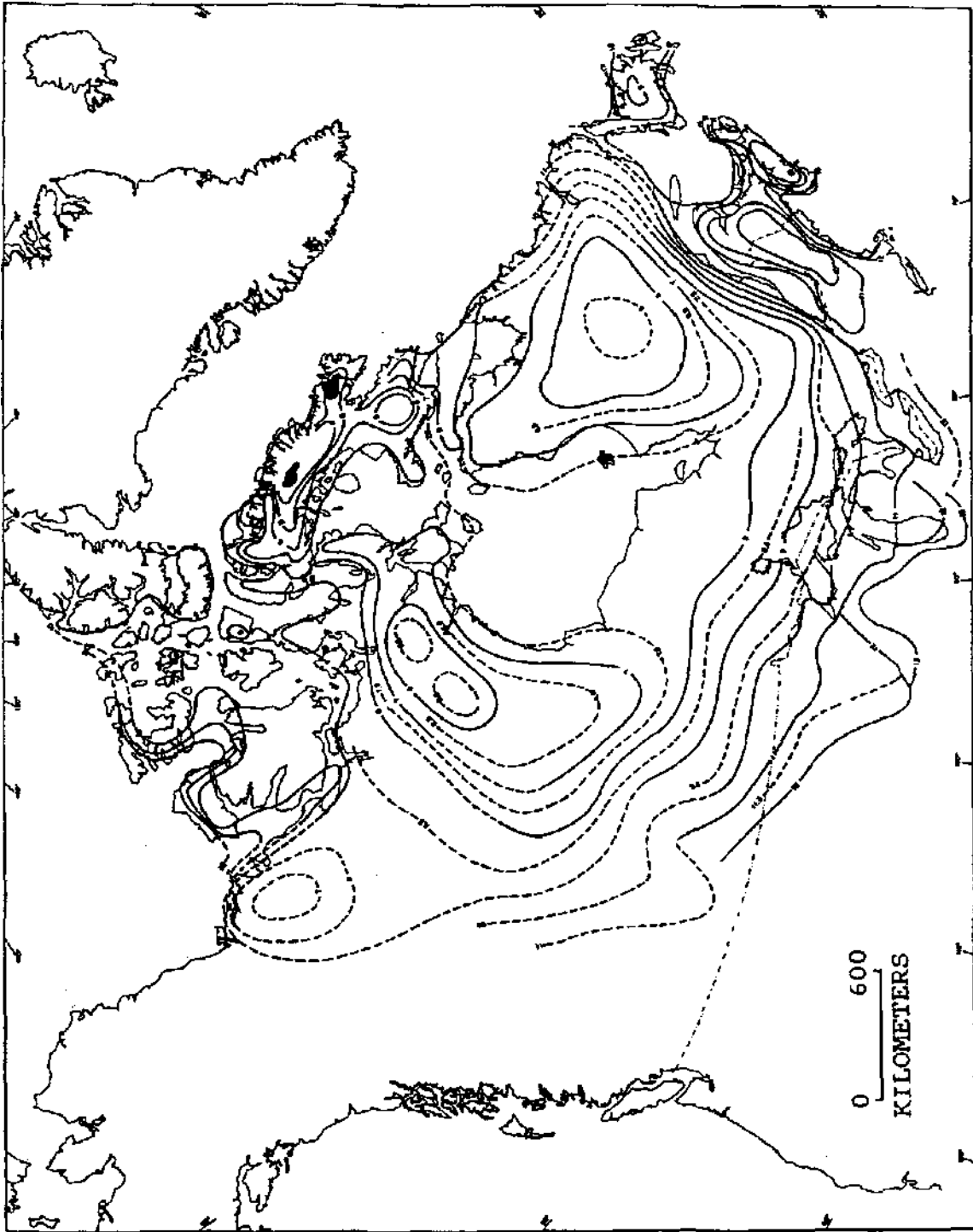




of sediments with which to create the deformable layer. Third, the northwest, southwest, and southern margins of the Laurentide Ice Sheet apparently retreated very rapidly at the end of the Wisconsinan (Figure 27) (Andrews, 1973; Bryson et al., 1969). Such rapid retreat is better explained by a thin ice sheet than by thick, modern ice sheets (Boulton and Jones, 1979, p 39). Fourth, Beget (1987, p 81) has mapped glacier thickness in Alaska where the northwest margin of the Laurentide Ice Sheet abutted against the Richardson and British mountains. Late Wisconsinan ice thicknesses were only 300 m, more than 150 km from the terminus, reflecting a deformable bed substrate. And finally, Bluemle et al. (1991a) have studied long, narrow drumlins (average length to width ratios of 30:1 to 50:1) near Velva, North Dakota. They concluded that these unique drumlins indicated thin, swiftly moving ice on a deformable bed characterized by high pore-water pressure (Bluemle et al., 1991a, p 47-48).

It has been previously noted that one of the common characteristics of low-profile ice sheets is that they form along the margins of large ice sheets. Boulton and Jones (1979) recognized three main zones that correspond to continental glaciation. First is the inner core zone, which is characterized by ice caps that have persisted throughout the Quaternary. The second is the intermediate zone, an area where ice sheets advance at the beginning of any expansion event. Third is the outer zone, where ice exists only during the coldest phases of the glacier growth. Because glaciers are characterized by net erosion beneath active ice and net deposition along the margins of the ice, the outer zone tends to be a zone of deposition of

Figure 27 - Isochrons marking the retreat of the Laurentide Ice Sheet at the end of the Wisconsinan. Note the rapid retreat indicated by the widely spaced contours southwest of Lake Superior. Contour interval varies between 500 and 1000 years. (Andrews, 1970, p 20).



N

unconsolidated sediments. Therefore, when the ice does extend into the outer zone, a large supply of material that can be saturated and deformed is readily available (Boulton and Jones, 1979, p 39-40). This is especially true if it is clayey (Clayton et al., 1985, p 239).

Absolute minimum ice thickness in the Lake Agassiz Basin has already been determined; the ice had to be thick enough to account for the 95 m of depression indicated by the Herman strandline and the Lake Agassiz sediments. However, maximum ice thickness is another matter. Whichever method is used, several assumptions have to be made. Often these assumptions are known to be wrong or unlikely (e.g., Mathews' assumption that the same parabolic profile exists for both the transverse and longitudinal slopes of the ice lobe), but they must be made either for the sake of simplifying the model or because better data are not available. Other times, the assumptions cannot be proven or compared to modern equivalents (e.g., the theory that deformable beds in the marginal areas of certain Wisconsinan ice sheets led to thin, elongate lobes), but they best explain the observed evidence.

When the estimates for maximum ice thickness at the international border were compared, the strandline and lake sediments method (1,040 m) gave a value similar to the basal shear strength method (986 m); the values differed by only 54 m. Minimum ice thickness, as calculated by the strandline and the effects of the lake sediments, was 280 m at Grand Forks.

In reality, ice thickness was most likely somewhere between these estimates because restrained rebound occurred during the retreat of the ice sheet, but the lake

water and sediments retarded rebound. Mathews\* method indicates about 424 m of ice at the international border and 390 m at Grand Forks (Table 6). Despite the problems with this method, it does provide an intermediate value. In addition, Andrews (1970, p 117) stated that present uplift in arctic Canada is approximately 0.5 to 0.8 m per 100 years and that final deglaciation occurred about 7,500 years B.P. In contrast, uplift in the southern part of the basin was approximately 1.0 m per 100 years when the Lake Agassiz Basin had been deglaciated for only about 1,000 years (Table 2). Apparently, rebound occurred more rapidly in the Lake Agassiz Basin than in arctic Canada.

This evidence leads to the conclusion that ice in the Lake Agassiz Basin during the Late Wisconsinan was much thinner than in arctic Canada. According to Mathews' equation and the results from the basal shear stress methods, ice thickness did not exceed a value of about 425 to 985 m at the present location of the international border. This caused a depression of approximately 140 to 330 m. This depression has rebounded completely if the mantle viscosity beneath the Lake Agassiz is less than  $2.53 \times 10^{20}$  Pa s; the strandlines indicate that mantle viscosity probably does not exceed  $9.6 \times 10^{19}$  Pa s. The depression produced a forebulge with a crest of 4.0 to 4.5 m, at a distance of about 4.8 km beyond the ice edge. Because the forebulge is an uplift of the crust, the beaches may have been raised slightly when they were formed, and then let down 4.0 to 4.5 m as the forebulge subsided after the melting of the ice. However, it seems unlikely that a forebulge of this magnitude would have had a major effect on the results obtained in this study, because the forebulge (4.0 to 4.5 m) is minor in comparison to the minimum amount of depression (95 m).

## Comparison With Nye's Method

### Introduction

. Nye (1957) proposed the following equation to calculate ice thickness:

$$(h / H)^{2+<lt;a> + (x / L)^{4<./m)} = 1 , \quad (18)$$

where h is the height of the upper surface of the ice at distance x from the center, H is the height of the ice at the center, L is the distance from the center to the edge of the ice sheet, and m is a constant, between 2 and 2.5.

### Calculations

Some calculations have been made in the Lake Agassiz Basin using the Nye equation to demonstrate the differences in the values obtained compared to the values from the marginal ice sheet methods. For these calculations, L is 2255 km (the distance between the center of the ice sheet near Hudson's Bay and Des Moines, IA), H is 4244 m (Sugden, 1977, p 27), and m is 2.25, as this is the average of the range assigned to m. Using these values, ice thickness was calculated to be 3000 m at Grand Forks and 3183 m at the international border (Table 11), about three times thicker than the values obtained using maximum basal shear stress and about 7.5 times thicker than by Mathews' method.

### Discussion

The values calculated by Nye's method differ considerably from thicknesses calculated using the marginal ice sheet methods, which give ice thickness values

Table 11 - Ice thickness as calculated from Nye's (1957) equation. The distance from the center of the ice sheet is the value used for L in the equation. The distance from the edge of the ice sheet allows for easy comparison with Table 4.

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Distance From Ice Center (km)	Distance From Ice Edge (km)	Ice Thickness (m)
2205	50	1036
2155	100	1373
2105	150	1618
2055	200	1816
2005	250	1985
1955	300	2134
1905	350	2268
1855	400	2390
1805	450	2503
1755	500	2607
1705	550	2704
1655	600	2795
1605	650	2881
1555	700	2963
1505	750	3040
1455	800	3113
1405	850	3183

---

ranging from 280 m to 1042 m at the International border (Table 7). This difference suggests that at least the margins of the Laurentide Ice Sheet behaved differently than existing ice sheets. The reason for this difference is the substrate; as has already been explained, modern ice sheets tend to rest on substrates with very high (100 to 150 kPa) basal shear stresses (Beget, 1987, p 82). The margins of the Laurentide Ice Sheet, on the other hand, had low (0.5 to 22 kPa) basal shear stresses (Beget, 1987, p 82; Clayton et al., 1985, p 239). The fact that Nye's equation works well on modern ice sheets but not on the marginal areas of many Wisconsinan ice sheets provides additional support for the deformable bed model.



## EFFECTS OF GLACIAL REBOUND ON THE LAKE AGASSIZ BASIN

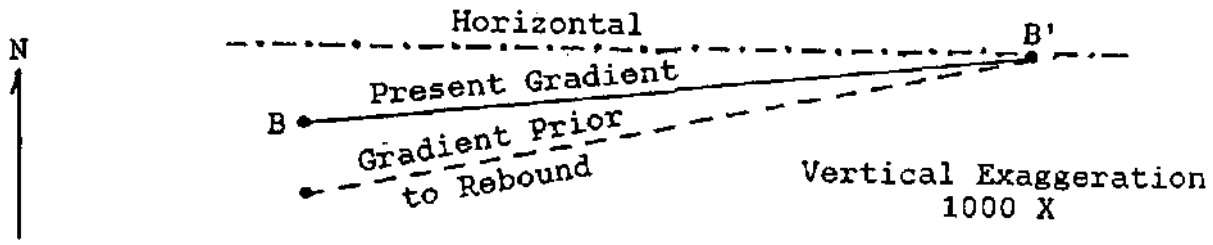
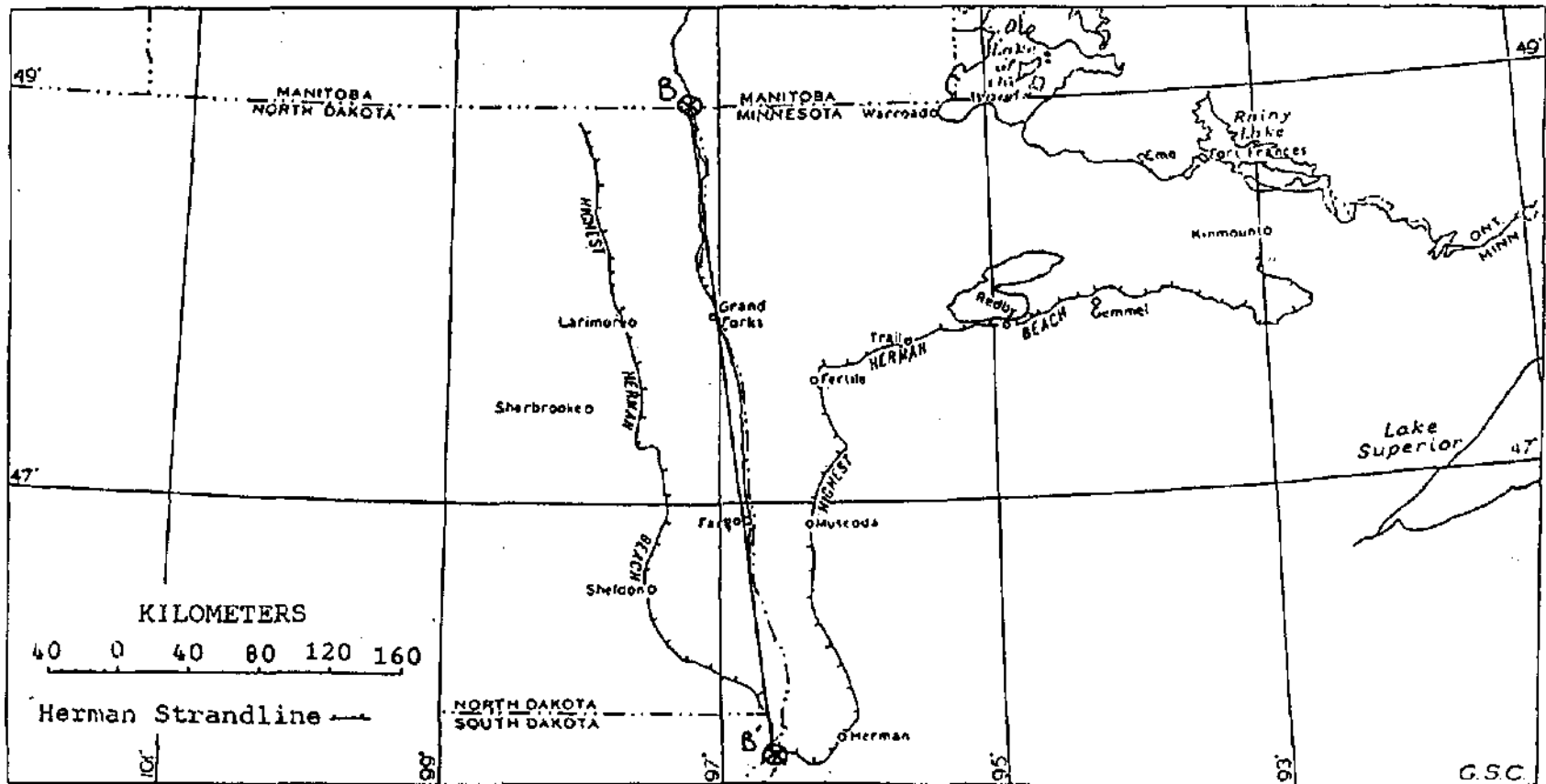
### Introduction

Rebound has affected geological processes in the Lake Agassiz Basin. The most obvious effect was the tilting of the Lake Agassiz beach ridges. But there are other effects as well, ones that have a greater influence on the human inhabitants of the basin.

### Decreased River Gradient

Because of greater rebound at the northern end of the Lake Agassiz basin, river gradients in the basin have been decreased. The northern end of the basin, at the international border, has been uplifted at least 54.5 m more than the southern end. From its head in southwestern Minnesota (which is only about 45 km southwest of the lowest point on the Herman strandline) to the international border, the Red River of the North is approximately 460 km long. This represents a decrease in gradient of at least 0.12 m/km (Figure 28), a significant decrease, especially considering that the present gradient of the Red River is only about 0.10 m/km between Grand Forks and Pembina (Harrison and Bluemle, 1980, p 9). It is this decrease in gradient that has led to the changes (e.g., changed river courses, frequent flooding, etc.) in the Lake Agassiz Basin associated with the Red River of the North and its tributary system.

Figure 28 - Map of southern Lake Agassiz; the current gradient of the Red River is shown in contrast to what the gradient probably was approximately 9,000 years B.P. when Lake Agassiz drained from the southern basin (map modified from Johnston, 1957, p 2).



### Changing River Courses

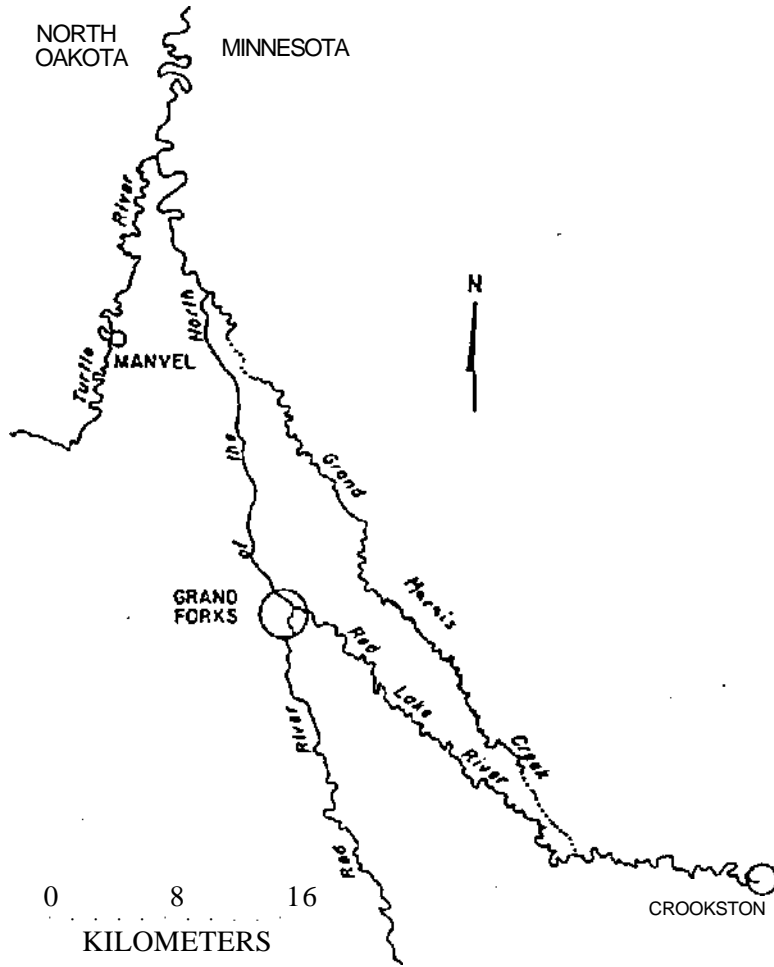
The isostatic rebound model for the Lake Agassiz Basin could be strengthened if there were evidence in addition to the tilted strandlines to indicate that isostatic rebound occurred there. A second line of evidence is found in the changes that the rebound caused to the routes of some Red River tributaries.

An example is the confluence of the Red and Red Lake rivers. Today, these two rivers converge at the city of Grand Forks, but at one time the confluence was about 32 km north of Grand Forks, nearly due east of Manvel (Figure 29) (Bluemle, 1991b, p 82). This is evidence for isostatic rebound; as rebound occurred the gradient decreased more rapidly at the northern end. Finally, when the gradient became too gentle for the Red Lake River to continue following its old channel, the river abandoned it to follow a new route with a steeper gradient, south of the old channel (Bluemle, 1991b, p 82).

By looking at Figure 29, it would be possible to imagine this example as being misinterpreted, that this was only an area where a small stream (Grand Marais Creek) happened to establish itself to the north of a larger river (the Red Lake River).

There are two major pieces of evidence other than the gradient that indicate isostatic rebound is responsible for this situation. First, Grand Marais Creek is a misfit stream, i.e., it is too small for the valley it occupies. Second, the Red River of the North between the Red Lake and Grand Marais confluences is considerably straighter than in other areas. This would indicate that the increased amount of water introduced to this section of the channel after the Red Lake River changed courses was

Figure 29 - The shift of the confluence of the Red Lake River and Red River of the North. The former confluence was where Grand Marais Creek enters the Red River. Note how straight the channel is between the two confluences (Bluemle, 1991, p 82).



too much for the earlier channel to carry. Thus, the meanders were washed out (i.e., the channel was straightened) until the Red River came to the Grand Marais confluence, where channel capacity was once again equal to the volume of water (Bluemle, 1991b, p 82). The migration of the channel therefore provides additional evidence in support of rebound.

#### Highly Meandering

Bluemle (1991b, p 81) contended that one of the most direct results of the decreased gradient is the highly meandering channel of the Red River of the North. He cited the numerous oxbow lakes and channel scars that have been formed by the Red River as evidence of a highly meandering river. Bluemle's contention was supported by Easterbrook (1993, p 127), who stated that meandering rivers are characterized by low gradients and banks with a high silt and clay content.

However, others contend that the Red River actually has a surprisingly narrow floodplain, few oxbow lakes and channel scars, and the high silt and clay content in the Red River's banks has actually decreased meandering because these sediments are so resistant to stream erosion (Reid, verbal communication, 1994). Schumm et al. (1987, p 273-274) found that a change in gradient did not necessarily correspond to an increase in meandering; other factors such as grain size play a significant role. Some researchers believe that meandering is controlled by the river's discharge (Petts and Foster, 1985, p 150); others believe that meandering is a mechanism for reducing excessive gradients (Richards, 1982, p 202), but in the Lake Agassiz basin the gradient

is already low. In short, to assume that meandering will increase because gradient decreased is oversimplifying the problem.

If the river meanders significantly more now than it did when it was originally formed, the banks could potentially be subjected to increased erosion (Easterbrook, 1993, p 122), a problem of great interest to an area that is largely agricultural. However, information gathered from several sources (Schumm et al., 1987; Petts and Foster, 1985; Richards, 1982) indicate that the Red River is not highly meandering, and that a simple change in gradient would not necessarily cause increased meandering. Furthermore, features associated with bank erosion, such as oxbow lakes and channel scars, are scarce, and the Red River has a low flow rate. Therefore, it also can be concluded that increased erosion related to greater meandering is probably not a problem in the Lake Agassiz basin.

#### Frequent Flooding

The most dramatic effect of the decreased gradient is seen when one of the rivers in the Lake Agassiz Basin floods. Flooding is a major concern because North Dakota's two largest metropolitan areas, Fargo (population 70,000) and Grand Forks (population 50,000), are along the banks of the Red River (Rand McNally, 1994, p 125). In addition, nearly 550,000 acres of agricultural land are in flood-prone areas of the basin (Harrison and Bluemle, 1980, p 8).

Due to the low gradient (which has decreased by differential isostatic rebound) stream velocities are low (Harrison and Bluemle, 1980, p 11). In addition, the highly

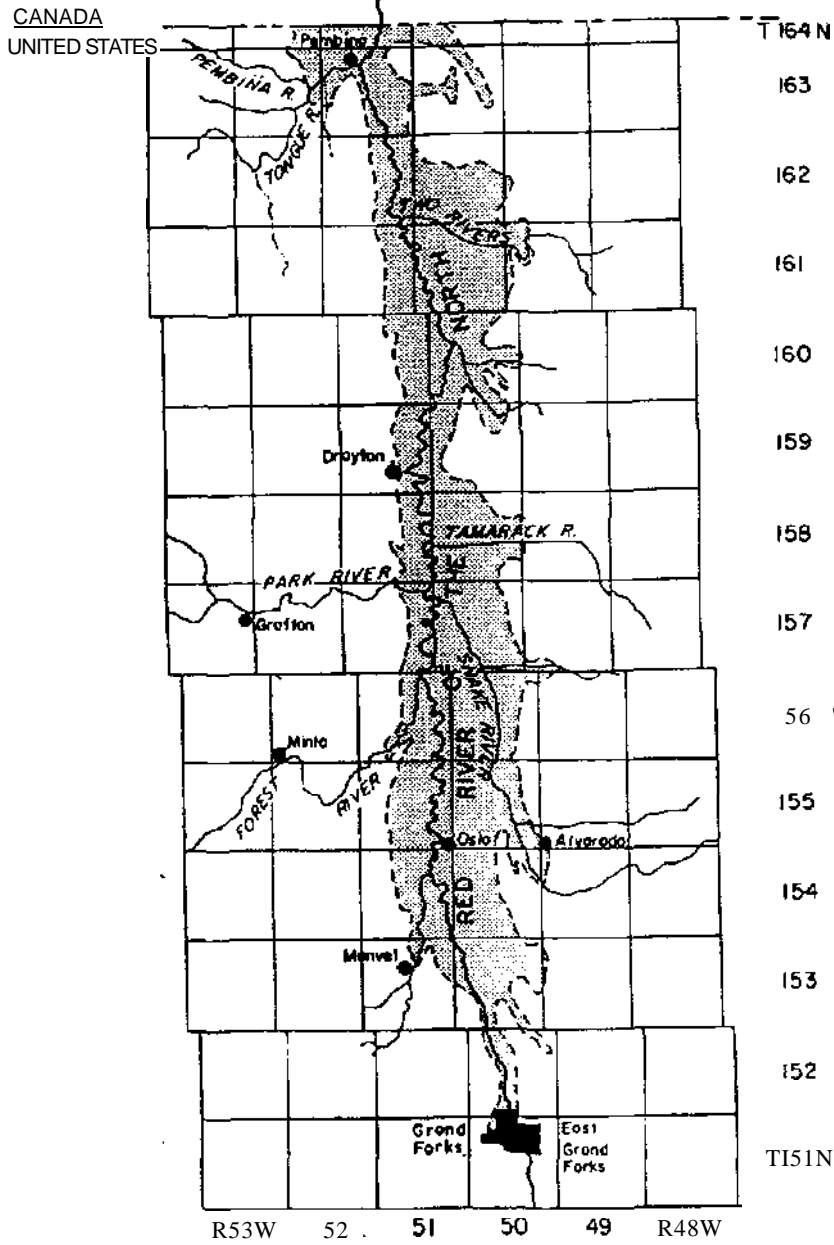


sinuous channels cannot carry large volumes of water as efficiently as a straighter channel (Bluemle, 1991b, p 81), and due to the low gradient the river can cut only a shallow valley (Harrison and Bluemle, 1980, p 9). Most importantly, the broad, flat floor of the basin allows floods to spread laterally for quite some distance (Bluemle, 1991b, p 81). This is particularly true of the area between Grand Forks and the international border, where, in 1950, parts of the Red River that have normal channel widths of about 30 m flooded areas up to 16 km wide (Figure 30) (Harrison and Bluemle, 1980, p 20). In addition, Schumm et al. (1985, p 272) found that rivers tend to respond to uplift with increased flooding. The decreased gradient of the basin has allowed more extensive flooding of the surrounding area.

Free-air gravity anomalies (Walcott, 1970) and Peltier's (1989) work indicate that isostatic rebound is not yet complete at the terminus of the Red River of the North, where the Nelson River enters Hudson's Bay. Studies by Silver and Chan (1988) and Pinet et al. (1991) suggest there is no asthenosphere beneath the Canadian Shield, in which case rebound would also still be occurring in the southern Lake Agassiz basin. It can be concluded, therefore, that there is potential for increased flooding problems in the southern Lake Agassiz basin in the future, as the average gradient continues to decrease.

Figure 30 - The area flooded by the Red River in 1950. Note the broad flood plain north of Grand Forks (Harrison and Bluemle, 1980, p 20).

III



/ ^I\*J Area Flooded

## CONCLUSIONS

Rebound in the southern Lake Agassiz Basin was retarded by the replacement of ice by water and sediments from Lake Agassiz. Because of this, rebound prior to the formation of the Herman strandline probably did not exceed 73% of total rebound. The uplift of the Herman strandline, when combined with the effects of the Lake Agassiz sediments, therefore, represents crustal depression of between 95 and 350 m, which correspond to ice thickness values of 280 to 1040 m, respectively.

The actual ice thickness in the southern Lake Agassiz Basin during the Late Wisconsinan was between the two extremes that have been calculated. Mathews' method and maximum basal shear stress indicate ice thicknesses of 425 to 986 m, only, reflecting a crustal depression of approximately 140 to 330 m.

The isostatic rebound in the basin has caused a decrease in the gradient of the Red River of the North. This decreased gradient has led to changes in the courses of some Red River tributaries and more frequent flooding in the basin. However, contrary to Bluemle's (1991b) conclusion, it is doubtful that the decreased gradient has significantly increased the meandering of the Red River.

Free-air gravity anomalies and evidence from the strandlines indicate that the crust in the southern part of the Lake Agassiz Basin has reached isostatic equilibrium. The current hingeline (i.e., the zero anomaly line on Figure 12a) for isostatic rebound extends through Lake Winnipeg to the north and through the Great Lakes to the east.

However, depending upon the presence or absence of asthenosphere beneath the southern Lake Agassiz basin, this may not be true. In either case, the ultimate outlet of the Red River (the point where the Nelson River flows into Hudson's Bay) is still rebounding. Therefore, the river's gradient will continue to decrease. This may increase the flooding problem in the southern part of the basin, as the average gradient continues to decrease.

## APPENDICES

APPENDIX A  
STRANDLINE ELEVATION DATA

## Herman strandline

<u>Quadrangle</u>	<u>Location</u>	<u>Grid Readings (m)</u>	
La Mars, ND-SD	Sec 32&33, T129N, R48W	329.2	327.6
		327.6	327.6
		327.6	327.6
		327.6	327.6
		327.6	327.6
		327.6	327.6
		327.6	327.6

Average: 327.8

Embden, ND	Sec 3, T138N R54W	329.2	333.7
		329.2	333.7
		329.2	333.7
		329.2	333.7
		329.2	333.7
		329.2	333.7
		329.2	333.7

Average: 331.5

Ayr NW, ND	Sec 7, T143N R53W	336.8	335.3
		336.8	335.3
		336.8	332.2
		336.8	332.2
		336.8	332.2
		333.7	332.2
		332.2	332.2

Average: 334.4

Inkster, ND	Sec 16, T154N R55W	358.1	350.5
		358.1	350.5
		358.1	350.5
		358.1	350.5
		358.1	350.5
		358.1	350.5
		358.1	350.5

Average: 354.3



## Herman strandline (continued)

Edinburg, ND	Sec 26, T158N R56W	365.7	371.8
		365.7	371.8
		365.7	371.8
		374.9	371.8
		374.9	371.8
		374.9	368.8
		374.9	374.9

Average: 371.4

Vang, ND	Sec 32, T164N R57W	384.0	381.0
		384.0	381.0
		384.0	365.7
		384.0	374.9
		384.0	374.9
		384.0	374.9
		384.0	374.9

Average: 379.7

Campbell strandline

<u>Quadrangle</u>	<u>Location</u>	<u>Grid Readings (TO)</u>	
La Mars, ND-SD	Sec 30, T129N R48W	303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3

Average: 303.3

Embden, ND	Sec 9, T138N R53W	303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3
		303.3	303.3

Average: 303.3

Inkster, ND	Sec 1, T154N R55W	304.8	309.4	
		304.8	309.4	
		^	304.8	309.4
		304.8	307.8	
		304.8	307.8	
		304.8	307.8	
		304.8	307.8	

Average: 306.7

Edinburg, ND	Sec 29, T158N R55W	310.9	317.0
		310.9	317.0
		310.9	317.0
		310.9	317.0
		310.9	317.0
		310.9	317.0
		310.9	317.0

Average: 314.0

## Campbell (continued)

Walhalla, ND	Sec 13, T163N R57W	320.0	310.9
		320.0	310.9
		320.0	313.9
		320.0	313.9
		320.0	313.9
		320.0	313.9
		320.0	313.9

Average: 316.6

Vang, ND	Sec 28, T164N R57W	326.1	320.0
		326.1	320.0
		323.1	320.0
		323.1	320.0
		323.1	320.0
		323.1	320.0
		323.1	320.0

Average: 322.1

## Emerado strandline

<u>Quadrangle</u>	<u>Location</u>	<u>Grid Readings (m)</u>	
Emerado, ND	Sec 7, T151N R52W	275.8	275.8
		275.8	275.8
		275.8	275.8
		275.8	275.8
		275.8	275.8
		275.8	275.8
		275.8	275.8

Average: 275.8

Veseleyville, ND	Sec 20, T156N R54W	277.4	277.4
		277.4	277.4
		277.4	277.4
		277.4	277.4
		277.4	277.4
		277.4	277.4
		277.4	277.4

Average: 277.4

LeRoy, ND	Sec 29, T163N R55W	281.9	281.9
		281.9	281.9
		281.9	281.9
		281.9	281.9
		281.9	281.9
		281.9	281.9
		281.9	281.9

Average: 281.9

APPENDIX B  
BEMIS MORaine ELEVATION DATA

B) Toronto Quadrangle, SD

NW 1/4, Sec 11, T113N R49W

Elevations (m)

612.6	611.1	611.1	612.6	611.1	611.1	609.6
612.6	612.6	612.6	611.1	612.6	612.6	611.1
612.6	612.6	612.6	612.6	612.6	612.6	611.1
612.6	612.6	612.6	612.6	612.6	612.6	609.6
611.1	611.1	612.6	612.6	612.6	612.6	612.6
606.5	609.6	611.1	612.6	612.6	612.6	612.6
606.5	609.6	609.6	612.6	612.6	612.6	612.6

Average: 611.7

B') Albany Quadrangle, MN

SW 1/4, Sec 14, T126N R31W

Elevations (m)

384.0	388.6	388.6	388.6	385.6	382.5	381.0
384.0	388.6	390.1	390.1	387.1	387.1	384.0
384.0	390.1	396.2	394.7	390.1	384.0	384.0
384.0	387.1	390.1	393.2	387.1	385.6	381.0
384.0	388.6	393.2	393.2	387.1	384.0	377.9
384.0	385.6	387.1	387.1	387.1	384.0	377.9
384.0	384.0	387.1	387.1	387.1	384.0	377.9

Average: 386.5

B and B' correspond to the locations shown on Figure 20 in the text, and are 200 km apart. The elevation at Des Moines is 300 m (Mathews, 1974). Des Moines is 400 km from B and B\

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